An Estimation Fusion Method for Including Phasor Measurements Into Power System Real-Time Modeling

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***Abstract*—This paper introduces a novel multistage state estimation architecture aimed at including synchronized phasor measurements into power system state estimation. The proposed architecture keeps unchanged the internal structure of existing SCADA-based estimators, so that phasor measurements are separately processed by a distinct estimation module. An additional stage makes use of principles from estimation fusion theory in order to combine SCADA- and PMU-based estimates. The proposed multistage scheme improves the quality of the estimates provided by the SCADA-based estimator and, under certain observability and correlation conditions, provides the same optimal results given by a hybrid simultaneous estimator (that is, an estimator that simultaneously processes both SCADA and PMU measurements). The separate processing of conventional and phasor measurements circumvents the technical challenge of conciliating, within the same estimation structure, data obtained from different measuring channels and gathered at very distinct sampling rates. In addition, the execution times tend to be significantly less than those required by hybrid simultaneous schemes. The paper describes in detail the multistage structure of the proposed state estimator and also addresses the benefits brought about by phasor measurement processing to state estimation accuracy. Results of several case studies conducted on the IEEE 57-bus, 118-bus, and 300-bus benchmark systems are used to illustrate the features of the proposed strategy.**

***Index Terms*—Estimation fusion, power system real-time modeling, power system state estimation, synchronized phasor measurements.**

I. Introduction

DURING the last decades, power system state estimation (PSSE) has established itself as the basic tool for real-time modeling of large electric power networks. As emerging Smart Grid concepts expand previous paradigms for power system operation and control, PSSE must evolve to keep pace with current trends [1]. This implies the incorporation of novel concepts and methods, such as those related to synchronized phasor measurement technology, which makes it possible to accurately measure bus voltage and branch current synchrophasors, something previously infeasible with conventional SCADA systems. As a consequence, the use of phasor measurement units (PMUs) in PSSE has deserved much attention in recent years [2]–[8].

In spite of the undisputed advantages of using synchrophasors for power system real-time modeling, it is unlikely that existing SCADA systems be entirely superseded by the PMU technology in the short run. Reasons for this include the still limited number of synchrophasor units deployed in real systems worldwide, which in most cases is insufficient to provide full system observability. In addition, one must take into account the significant investments made in the past to build complex SCADA infrastructures that should not be simply discarded. Hence, one envisages a period of several years during which both technologies will coexist.

As far as PSSE is concerned, the above scenario leads to the challenge of how to conceive estimation strategies able to benefit from both technologies. A straightforward solution is to devise state estimators able to simultaneously process both SCADA and PMU measurements. This scheme is referred to as *hybrid simultaneous state estimation* [5], [9].

However, the simultaneous approach faces serious practical obstacles. The most evident is the need to introduce significant changes to existing state estimation software. Moreover, PMU measuring channels are distinct from those used by SCADA, and meters are sampled at very different rates. Previous research efforts have also recognized those challenges [2], [4], [5] and proposed alternative estimation schemes. All of them entail the decoupling of the estimation process into two stages, each of them devised to process each measurement type (SCADA or PMU).

This paper proposes a novel state estimation architecture based on Estimation Fusion methods which exhibits a number of desirable properties. First of all, the proposed strategy does not impose any change to the structure of existing SCADA-based state estimators (which we refer to as SSE). Instead, it is considered as one of the estimation modules of a multistage structure, which also comprises a PMU-based state estimation module (or PSE) that is devised to process only phasor measurements. The resulting SSE and PSE estimates are then combined, or fused, in a subsequent Fusion stage, producing a final vector of estimates. The algorithm underlying the Fusion stage is based on Multisensor Data Fusion Theory, a relatively recent research field related to the Aerospace, Information Theory and Signal Processing areas [10]. In this paper, we apply to PSSE the estimation fusion methods presented

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in detail in [11]–[13]. We also make use of theoretical results developed in connection with other research efforts in the area, such as those reported in [14]–[16].

Additional features of the proposed strategy are: 1) no restrictions are imposed to the algorithm of the PSE estimator, which is completely independent of its SCADA counterpart and can, for instance, be based on a linear measurement model; 2) observability with respect to phasor measurements is not assumed since, as discussed in Section V-C, PMU-unobservability can be circumvented at the PSE stage through the use of *a priori* state information; 3) last but not least, the Fusion stage relies on a minimum variance criterion, which produces an unbiased, minimum variance final estimate [14]. That is to say, in the absence of gross measurement errors, the state estimates provided by the fusion approach are free of bias and present minimum deviation from the true state values. In this paper, we also show that the computational issues related to the fusion stage are efficiently resolved by using techniques and tools familiar to power system engineers, such as Gauss elimination and sparsity.

Section II of this paper briefly reviews the conventional PSSE background. The distinct strategies to include phasor measurements into PSSE are detailed in Section III. The principles of estimation fusion methods are presented in Section IV, followed by their proposed application to PSSE, in Section V. Results of several case studies performed on the 57-bus, 118-bus, and 300-bus IEEE benchmark systems are presented and analyzed. Finally, Section VII provides the concluding remarks.

II. State Estimation Background

A. Weighted Least Squares and Gauss-Newton Method

Power system state estimation provides state estimates by processing real-time data gathered from remote meters. Consider that a set of m measurements is taken on an N-bus power network. The nonlinear measurement model that relates measurements and state variables is given by

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \boldsymbol{\varepsilon} \tag{1}$$

where $\mathbf{z}$ is the $m \times 1$ measurement vector, $\mathbf{x}$ is the $n \times 1$ vector of state variables to be estimated, $\mathbf{h}(\mathbf{x})$ is the $m \times 1$ vector of nonlinear functions relating measured quantities and state variables, and $\boldsymbol{\varepsilon}$ is the $m \times 1$ measurement error vector, whose $m \times m$ covariance matrix is denoted by $\mathbf{R}$. The number of state variables is given by $n = 2N - 1$. Measurements are usually assumed uncorrelated, so that $\mathbf{R}$ is diagonal, with its ith diagonal entry equal to σ_i^2, which is the variance of the error of measurement i.

The weighted least-squares (WLS) formulation to the PSSE problem is based on the minimization of the weighted sum of the squared residuals

$$\min_{\hat{\mathbf{x}}} J(\hat{\mathbf{x}}) = \frac{1}{2}[\mathbf{z} - \mathbf{h}(\hat{\mathbf{x}})]^t \mathbf{R}^{-1} [\mathbf{z} - \mathbf{h}(\hat{\mathbf{x}})]. \tag{2}$$

The above problem is solved through the Gauss-Newton method, leading to an iterative process in which the so-called *normal equation* is solved in each iteration [17]–[19]:

$$\mathbf{G}\boldsymbol{\Delta}\mathbf{x} = \mathbf{H}^t \mathbf{R}^{-1} \boldsymbol{\Delta}\mathbf{z} \tag{3}$$

where $\mathbf{G} = (\mathbf{H}^t\,\mathbf{R}^{-1}\,\mathbf{H})$ is referred to as gain matrix; $\mathbf{H}$ is the $m \times n$ Jacobian matrix of $\mathbf{h}(\mathbf{x})$ computed at a given point $\mathbf{x}^k$, and $\boldsymbol{\Delta}\mathbf{z} = \mathbf{z} - \mathbf{h}(\mathbf{x}^k)$.

The solution of (3) yields vector $\boldsymbol{\Delta}\mathbf{x}$ of increments to the states, so that the updated state vector is obtained as

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \boldsymbol{\Delta}\mathbf{x}. \tag{4}$$

The convergence of the iterative procedure is attained when $\boldsymbol{\Delta}\mathbf{x}$ becomes smaller than a pre-specified tolerance.

It is often desirable to assess the level of confidence on the accuracy of the state estimates. This is provided by the covariance matrix of the estimation errors [17], given by

$$\mathbf{P} = \mathbf{G}^{-1} = (\mathbf{H}^t\,\mathbf{R}^{-1}\,\mathbf{H})^{-1}. \tag{5}$$

B. A Priori State Information

A priori information consists of additional knowledge on the state variables available prior to the execution of state estimation. It is usually possible to assign a degree of confidence to such data, under the form of a covariance matrix. *A priori* state values contribute to state estimation in a similar fashion as measured data, although their accuracies, as given by the the corresponding variances, are usually less than the telemetered ones. In PSSE, *a priori* information is sometimes used to circumvent observability problems, such as those that occur in topology error identification studies [20].

If $\bar{\mathbf{x}}$ denotes the vector of *a priori* state values and $\mathbf{P}_0$ is the $n \times n$ corresponding covariance matrix, *a priori* information can be embedded into the WLS problem by augmenting the objective function (2) as [21]

$$\min_{\hat{\mathbf{x}}} J(\hat{\mathbf{x}}) = \frac{1}{2}[\mathbf{z} - \mathbf{h}(\hat{\mathbf{x}})]^t \mathbf{R}^{-1} [\mathbf{z} - \mathbf{h}(\hat{\mathbf{x}})] + \frac{1}{2}(\hat{\mathbf{x}} - \bar{\mathbf{x}})^t \mathbf{P}_0^{-1} (\hat{\mathbf{x}} - \bar{\mathbf{x}}). \tag{6}$$

The optimality conditions for Problem (6) lead to the following extended version of the normal equation [21]:

$$\left[\mathbf{H}^t\,\mathbf{R}^{-1}\,\mathbf{H} + \mathbf{P}_0^{-1}\right] \boldsymbol{\Delta}\mathbf{x} = \mathbf{H}^t\,\mathbf{R}^{-1} \boldsymbol{\Delta}\mathbf{z} + \mathbf{P}_0^{-1} \boldsymbol{\Delta}\overline{\mathbf{x}} \tag{7}$$

where $\boldsymbol{\Delta}\overline{\mathbf{x}} \triangleq (\bar{\mathbf{x}} - \mathbf{x}^k)$.

C. Including A Priori Information in PSSE Through Orthogonal Estimators

Orthogonal methods based on Givens rotations have been originally proposed as alternative solvers for WLS estimators in order to avoid numerical instability problems that may occur in the solution of normal equations [21]–[23]. In addition to that, however, some variants of Givens rotations-based state estimators, such as the 3-multiplier version [22], exhibit some interesting properties that facilitate the processing of *a priori* state information. In fact, it is shown in [20] that those estimators are able to take *a priori* information into account at the initialization stage, so that no extra computational cost is incurred.

In this paper, we take advantage of the above described property in order to get around observability problems that arise at the PMU-based state estimation stage, as discussed in Section V-C.

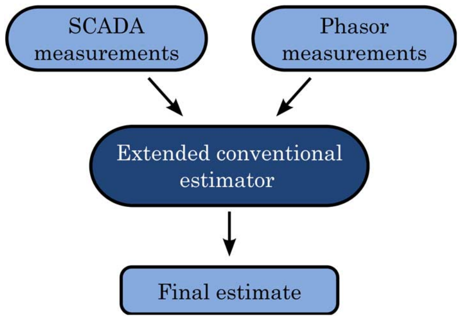


Fig. 1. Centralized estimator.

III. Phasor Measurements in Power System State Estimation

During the 1980s, the advent of the Global Positioning System (GPS) and improvements on synchronizing techniques made it possible the acquisition of measurements under the same time reference at geographically distant locations of electric power networks [24]. Those principles led to the development of the so-called Synchronized Phasor Measurement Systems, composed of PMUs installed at substations of the power network. Through such systems, reliable phasor measurements of magnitude and phase angle of nodal voltages and branch currents become available at high sampling rates to several power system applications. Real-time power system modeling, and PSSE in particular, are among those functions which can draw significant benefits from the availability of PMU measurements.

However, the incorporation of PMU data into PSSE poses a number of challenges. First of all, the penetration of synchrophasor units in power system is still limited, and dramatic changes in such scenario are not expected in the near future. As a consequence, available phasor data must be combined with conventional SCADA measurements in order to perform PSSE. This, in turn, raises other hardware and software challenges connected to issues such as: the very distinct sampling ratios with which PMU and SCADA measurements are gathered, the integration of distinct measurement channels, and the fact that existing EMS software is not prepared to process phasor variables. Alternative forms to include PMU data into PSSE are discussed in the remaining of this section.

A. Hybrid Simultaneous State Estimation

The straightforward solution to include PMU data into PSSE is to devise state estimators able to simultaneously process both SCADA and PMU measurements. Such a strategy is referred to in this paper as *hybrid simultaneous state estimation (HSSE),* and is illustrated in Fig. 1. The HSSE strategy has been widely investigated [7], [9], [25]–[27] and in theory exhibits very good results. In fact, an important property of this strategy is its joint optimality, in the sense that, by processing all available SCADA and PMU measurements by the same estimator, the best solution that fits the hybrid measurement set is obtained. However, a number of practical issues may undermine the adoption of the hybrid simultaneous solution, such as the need to conciliate the great difference in sampling ratios of PMU and SCADA data, and the fact that existing EMS software must undergo significant changes in order to accommodate phasor data processing. A possible alternative to handle the distinct features of both technologies is to devise novel state estimation architectures according to which SCADA and PMU measurement subsets are respectively processed by a distinct estimation module able to recognize the properties of each subset. This alternative approach is discussed in the sequel.

B. Multistage State Estimation

In order to cope with the practical difficulties that afflict the hybrid simultaneous solution, state estimation structures composed by two or more stages can be devised, in such a way that SCADA and PMU data are processed by separate estimation modules. The integration of the estimates produced by the individual modules is either embedded in the second stage, along with the processing of the phasor measurements, or carried out in a third independent stage, depending on the conception of the estimation architecture. Examples of two-stage hybrid estimators of the former type are the scheme proposed in [2], which treats the SCADA-based estimates produced by the first stage as pseudo-measurements, jointly processed with PMU data in the second stage; and the structure presented in [4], in which the SCADA-based estimates are considered as *a priori* information by an orthogonal estimator that processes the PMU measurements. A minor, intermediate step may be needed for conversion from polar to rectangular coordinates, in order to take advantage of the rectangular formulation linearity.

A definite advantage of the architectures proposed in [2] and [4] is that they do not require any change in existing SCADA-based state estimators. On the other hand, both schemes rely on simplifying assumptions concerning statistical properties of the final estimates. In addition, the decoupling of the estimation process into two stages does not provide the optimal solution.

In this paper, a novel multistage estimation architecture is proposed whose main feature is the ability to preserve the optimality of the simultaneous state estimation solution, under certain conditions, which are discussed in Section IV-C. It also maintains the desirable properties of two-stage strategies, while avoiding its simplifying assumptions. The proposed estimator relies on Estimation Fusion theory, which is briefly reviewed in the next section.

IV. Estimation Fusion Methods

A. Concepts From Multisensor Data Fusion Theory

Multisensor Data Fusion (MDF) theory is a relatively recent research and development field related to the Aerospace, Information Theory and Signal Processing area [10], and deals with processes or environments monitored by distinct classes of sensors. The sensors pertaining to each of such classes may share common characteristics, such as the underlying technology, level of accuracy, etc. The problem then arises as how to combine the data generated by the various sensors in an optimal manner, in the sense that the quality of the knowledge so produced about the monitored process is improved with respect to the one obtained from a single sensor class. This concept of gathering and combining information from several sources to improve those available with a single sensor class can be borrowed by PSSE, aiming at combining (or *fusing*) SCADA and phasor data. In this context, a branch of MDF is

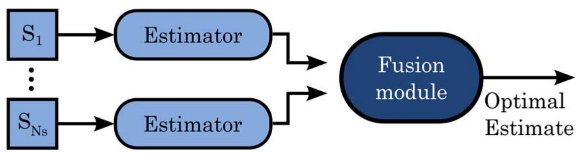


Fig. 2. Decentralized estimator.

of particular interest: *decentralized estimation fusion*, that is concerned with combining *processed* data generated by distinct sets of sensors [10], [12], [14]. Fig. 2 illustrates that particular fusion architecture.

The following subsections summarize the multisensor data fusion mathematical formulation and the main results presented in [11]–[13]. Attention is then focused on the two sensor problem, specifically addressed in [16]. Some theoretical results derived in [15] are also employed.

B. Mathematical Formulation of the Decentralized Estimation Fusion Problem

Consider that a particular process is monitored by N_s distinct sets of sensors. Based on the data available from each set, we assume that a $n \times 1$ vector of estimates $\hat{\mathbf{x}}_i, i = 1, \ldots, N_s$, is obtained for the state variables of the process. In addition, the resulting estimation errors can be correlated, so that the corresponding $n \cdot N_s \times n \cdot N_s$ covariance matrix is given by

$$\mathbf{P} = \begin{bmatrix} P_{11} & \cdots & P_{1N_s} \\ \vdots & \ddots & \vdots \\ P_{N_s1} & \cdots & P_{N_sN_s} \end{bmatrix}. \tag{8}$$

The optimal estimation fusion problem is formulated as a particular linear combination of the individual estimates $\hat{\mathbf{x}}_i$, that is

$$\hat{\mathbf{x}}^* = \mathbf{W}_1^t \hat{\mathbf{x}}_1 + \cdots + \mathbf{W}_{N_s}^t \hat{\mathbf{x}}_{N_s} \triangleq \mathbf{W}^t \hat{\mathbf{x}}_a \tag{9}$$

where $\mathbf{W}_1, \ldots, \mathbf{W}_{N_s}$ are $n \times n$ weighting matrices, $\mathbf{W} \triangleq [\mathbf{W}_1^t, \ldots, \mathbf{W}_{N_s}^t]^t$, and $\hat{\mathbf{x}}_a = [\hat{\mathbf{x}}_1^t, \ldots, \hat{\mathbf{x}}_{N_s}^t]^t$. The $\mathbf{W}_i$ weighting matrices are obtained by solving the following optimization problem:

$$\begin{aligned} \min_{\mathbf{W}} \quad & E[(\mathbf{W}^t \hat{\mathbf{x}}_a - \mathbf{x})(\mathbf{W}^t \hat{\mathbf{x}}_a - \mathbf{x})^t] \\ \text{s. to} \quad & \sum_{i=1}^{N_S} \mathbf{W}_i = \mathbf{I} \end{aligned} \tag{10}$$

where $E[\cdot]$ is the expectation operator, $\mathbf{x}$ is the vector of true values for the process state variables and $\mathbf{I}$ is the $n \times n$ identity matrix. Therefore, Problem (10) aims at minimizing the covariance of the estimation error $(\hat{\mathbf{x}}^* - \mathbf{x})$. This estimator is therefore referred to as *Linear Unbiased Minimum Variance, or Linear Minimum Mean Square Error (LMMSE)*, or *Best Linear Unbiased Estimation (BLUE) Estimator [14], [15]*. If the covariance matrix $\mathbf{P}$ in (8) is nonsingular, it can be shown that the solution of Problem (10) is unique, and the weighting matrices $\mathbf{W}_i$ are given by [11]

$$\mathbf{W}_i = \left(\sum_{k=1}^{N_s} \mathbf{P}_{ik}^{-1} \right) \left(\sum_{j,k=1}^{N_s} \mathbf{P}_{jk}^{-1} \right)^{-1}, i = 1, \ldots, N_s. \tag{11}$$

The case when only two classes of sensors are considered is of particular interest in this paper. For $N_s = 2$, (9) yields

$$\hat{\mathbf{x}}^* = \mathbf{W}_1^t \hat{\mathbf{x}}_1 + \mathbf{W}_2^t \hat{\mathbf{x}}_2 \tag{12}$$

and the weighting matrices are obtained from (11). For the particular case of only two classes of sensors, those expressions can be considerably simplified, as shown in [12, p. 172] and also in [13, Appendix]. Accordingly, the following optimal estimate results:

$$\begin{aligned} \hat{\mathbf{x}}^* = & (\mathbf{P}_{22} - \mathbf{P}_{21})(\mathbf{P}_{11} + \mathbf{P}_{22} - \mathbf{P}_{12} - \mathbf{P}_{21})^{-1} \hat{\mathbf{x}}_1 \\ & + (\mathbf{P}_{11} - \mathbf{P}_{12})(\mathbf{P}_{11} + \mathbf{P}_{22} - \mathbf{P}_{12} - \mathbf{P}_{21})^{-1} \hat{\mathbf{x}}_2. \end{aligned} \tag{13}$$

Equation (13) is referred to as *Bar-Shalom-Campo fusion formula*, and it is applicable to the particular case of two classes of sensors [16]. Moreover, if the individual estimates $\hat{\mathbf{x}}_1$ e $\hat{\mathbf{x}}_2$ can be assumed as uncorrelated, the above fusion formula becomes

$$\hat{\mathbf{x}}^* = \mathbf{P}_{22}(\mathbf{P}_{11} + \mathbf{P}_{22})^{-1} \hat{\mathbf{x}}_1 + \mathbf{P}_{11}(\mathbf{P}_{11} + \mathbf{P}_{22})^{-1} \hat{\mathbf{x}}_2. \tag{14}$$

C. Optimality of the Decentralized Fusion Estimator

An important theoretical property of the Decentralized Fusion Estimator that aggregates N_s estimators as described in the previous subsection is that, under certain conditions to be detailed next, its results are basically the same produced by a centralized (that is, a hybrid simultaneous) estimator that jointly processes the whole set of measurements made available by the N_S sets of sensors. That amounts to saying that there is no performance degradation incurred by adopting the decentralized strategy, as long as those conditions are satisfied.

Specifically, the conditions for the optimal decentralized and centralized fusion estimators to exhibit the same performance are [15]: 1) the *measurement errors are uncorrelated across sensor sets*, and 2) the *observation matrices have full column rank*. The term "observation matrix" refers to the matrix which relates the measurement and state vectors in a linear measurement model; it corresponds to the Jacobian matrix in the PSSE problem.

In the hybrid SCADA and PMU state estimation context, the column rank requirement above implies that the electrical network must be both SCADA- and PMU-observable. On the other hand, the error uncorrelation condition would be granted by assuming that SCADA and PMU metering channels are independent. Those issues are further discussed in the next section.

V. Estimation Fusion Applied to SCADA- and PMU-Based PSSE

A. Fusion of SCADA- and PMU-Based Estimates

In this paper, we propose the application of Multisensor Data Fusion concepts to incorporate PMU measurements into PSSE. It is based on the interpretation of SCADA and phasor measuring systems as distinct classes of sensors in charge of monitoring the same process, which is the power network. Accordingly, each of those monitoring systems processes its own measurement set in order to produce independent state estimates that reflect the current operating condition of the power system. We refer to SCADA-based and PMU-based state estimators as SSE and PSE, respectively. We also denote by $\mathbf{z}_S$ and $\mathbf{z}_P$ the measurement vectors whose processing by SSE and PSE, respectively, produces states estimates $\hat{\mathbf{x}}_S$ and $\hat{\mathbf{x}}_P$. Finally, the corresponding error covariance matrices are denoted by $\mathbf{P}_S$ and $\mathbf{P}_P$. The latter can be computed as given by (5) by applying indices S or P to refer to the appropriate Jacobian and measurement error covariance matrices.

Assuming that individual estimates $\hat{\mathbf{x}}_S$ and $\hat{\mathbf{x}}_P$ are available, along with their respective error covariance matrices, our objective is to determine how they can be optimally combined in the Fusion Module of Fig. 2. For this purpose, we also make use of the reasonable assumption that the SCADA and PMU metering channels are independent, so that the respective estimation errors are uncorrelated. As a consequence, (14) applies, yielding

$$\hat{\mathbf{x}}^* = \mathbf{P}_P(\mathbf{P}_S + \mathbf{P}_P)^{-1}\hat{\mathbf{x}}_S + \mathbf{P}_S(\mathbf{P}_S + \mathbf{P}_P)^{-1}\hat{\mathbf{x}}_P. \quad (15)$$

Since the expression $(\mathbf{P}_S + \mathbf{P}_P)^{-1}$ is common to both terms in the rhs of (15), it is interesting to notice that the matrix that actually defines the weight of each component estimate $\hat{\mathbf{x}}_S$ and $\hat{\mathbf{x}}_P$ of the optimal solution is the error covariance matrix associated with the estimate provided by the other component. Considering that less accurate estimates imply a covariance matrix with larger values, it follows that better quality estimates receive larger weights, as one would expect.

We henceforth refer to the estimator based on the above principle as *Fusion State Estimator* (FSE).

B. Computational Aspects

In the form presented in (15), the Bar-Shalom-Campo equation is not amenable for application to large networks, due to the explicit matrix inversion on its right-hand side. Fortunately, however, it is possible to develop an alternative form for that equation that prevents those computational difficulties. In order to do so, we go back to the original expressions for the optimal weighting matrices given by (11), and then apply the uncorrelation assumption regarding $\mathbf{z}_S$ and $\mathbf{z}_P$. If in addition we use (5) to define the SSE and PSE gain matrices $\mathbf{G}_S$ and $\mathbf{G}_P$, and also consider the fact that the covariance matrices are symmetric, (15) becomes

$$\hat{\mathbf{x}}^* = (\mathbf{G}_S + \mathbf{G}_P)^{-1}\mathbf{G}_S\hat{\mathbf{x}}_S + (\mathbf{G}_S + \mathbf{G}_P)^{-1}\mathbf{G}_P\hat{\mathbf{x}}_P \quad (16)$$

which can be rewritten as

$$(\mathbf{G}_S + \mathbf{G}_P)\hat{\mathbf{x}}^* = \mathbf{G}_S\hat{\mathbf{x}}_S + \mathbf{G}_P\hat{\mathbf{x}}_P. \quad (17)$$

Equation (17) can be solved by sparse triangular factorization and forward/back substitution, taking also advantage from the fact that gain matrices $\mathbf{G}_s$ and $\mathbf{G}_P$ are available from the individual SSE and PSE solutions. Notice also that the rhs vector consists of a weighted combination of SSE and PSE estimates.

C. PMU-Observability Issues

1) PMU-Unobservable Networks: A first-glance interpretation of (15) suggests that both SCADA-observability and PMU-observability constitute necessary conditions for applying the estimation fusion strategy, since those properties are required for computing the individual estimates to be fused, i.e., $\hat{\mathbf{x}}_S$ and $\hat{\mathbf{x}}_P$.

The assumption that the network is observable with respect to SCADA measurements is realistic, since existing SCADA-based state estimators employ metering schemes which are designed to provide observability even under stringent conditions. The same does not apply, however, to PMU-observability, since PMU penetration in power networks is still limited, and no dramatic changes concerning this are expected in the near future.

Fortunately, a closer look into the problem reveals that PMU-unobservability can be circumvented by incorporating complementary information into the PSE stage. Along with the existing phasor measurements, such complementary data are used to artificially provide observability and thus to allow the computation of $\hat{\mathbf{x}}_P$. Since that additional information is usually approximate and possibly inaccurate, care must be taken in order to avoid contaminating the estimates directly based on the PMU measurements produced by the PSE module. This can be accomplished by combining a judicious choice of variance values and the filtering properties of the state fusion process, as explained next.

The above mentioned complementary information can take different forms, pseudo-measurements being an obvious possibility. Nevertheless, in this paper we resort to *a priori* state information data, for two reasons: first of all, some kind of information on state variable values is always available, either as recently calculated state estimates or, lacking them, standard "expected" values for bus complex voltages. The second and most important reason is computational: as discussed in Section II, there are variants of Givens rotations-based estimators which process *a priori* information at initialization time, so that no computational cost is incurred.

Therefore, the PMU-unobservability issue is taken care of by assigning *a priori* state information to every PMU-unobservable bus of the network. What remains to be discussed is the impact of such approximate data on the quality of the final state estimates. In regard to that, two aspects are paramount: the criticality of such data, and the variances assigned to them. The former stems from the fact that the *a priori* data is essential to ensure observability; an important implication derives from the well known result that the residuals of critical data are zero [28], thus ensuring that the estimates for neighboring states will not be contaminated by inaccurate *a priori* values.

The second factor concerns the variances assigned to *a priori* state values. The larger degree of uncertainty associated to them must be accounted for when defining their variances, which compose matrix $\mathbf{P}_0$ of Section II-B. Thus, the latter are usually some orders of magnitude larger than the telemeasurement variances. At the outcome of the PSE stage, those large entries will be reflected on the error covariance matrix $\mathbf{P}_P$, whose diagonal values corresponding to the PMU-unobservable states, will be also large. Since large variances lead to small weighting factors at the fusion step, the estimates provided for those states by the PSE module receive very small weights, so that the corresponding SCADA-based estimates eventually prevail at the completion of the fusion stage. In other words, provided that their variances are properly defined, the effects of assumed *a priori* information are filtered out in the final fusion stage and have no significant effect on the optimal estimates. Such claim is indeed confirmed by the results presented in Section VI.

2) Observability × Optimality: Additional remarks are also in order concerning the optimality of the fusion-based multistage estimator when the power network is not PMU-observable. As discussed in Section IV-C, the performances of the proposed estimator and of a hybrid simultaneous one are essentially the same, provided that some conditions are fulfilled. Those include the observability with respect to every particular set of sensors involved in the fusion strategy. Whenever the observability conditions can only be granted through the use of pseudo-measurements or *a priori* information, as discussed in the previous subsection, some degradation in performance is to be expected. In order to measure such a performance degradation, we make use of the *Degree of Suboptimality*, μ_{SubOpt}:

$$\mu_{SubOpt} \triangleq \frac{J - J^*}{J^*} \times 100\% \tag{18}$$

where J^* is the weighted sum of the squared residuals computed for the hybrid simultaneous state estimator (HSSE), which theoretically yields the optimal solution, and J is the same quantity computed from the results provided by the multistage estimator. This index has been previously used in the literature to evaluate how much the quality of a sub-optimal strategy deviates from the optimal one [29].

In the next section, such an index is applied to assess the performance of the proposed strategy under distinct conditions.

D. Impact on Bad Data Analysis

This paper is mainly intended to apply the estimation fusion principles in order to incorporate phasor measurements into PSSE. Nevertheless, the impact of the fusion strategy on bad data processing deserves some attention, since error analysis is an essential attribute of any power system state estimator. Bad data analysis in the context of hybrid state estimation has been previously addressed in [6]. However, fusion-based estimators exhibit a distinct architecture, so that the bad data issue has to be re-examined. In this respect, two situations should be addressed, namely, gross errors in SCADA measurements and bad data among phasor measurements.

Bad data analysis for SCADA-based state estimators has been a matter of interest through many years, and a variety of methods for gross errors detection and identification are available [18], [19], [30]. A common characteristic of all those methods is the requirement of a certain level of local measurement redundancy to ensure good performance.

That knowledge can be transported to the multistage state estimation environment proposed in this paper by performing bad data analysis at the SSE module, through the application of reliable methods such as those described in [30], for instance. By that means, one would ensure that the SCADA-based estimates to be later fused with the results of the PSE module are free from the influence of bad data. However, performance still depends on the existence of adequate levels of local redundancy. Although that is usually granted for most regions comprising a given power network, there are often weak spots in which the available telemetered information is insufficient, so that critical measurements (whose gross errors are undetectable, [28]) and critical sets (whose gross errors are unidentifiable, [30]) may occur.

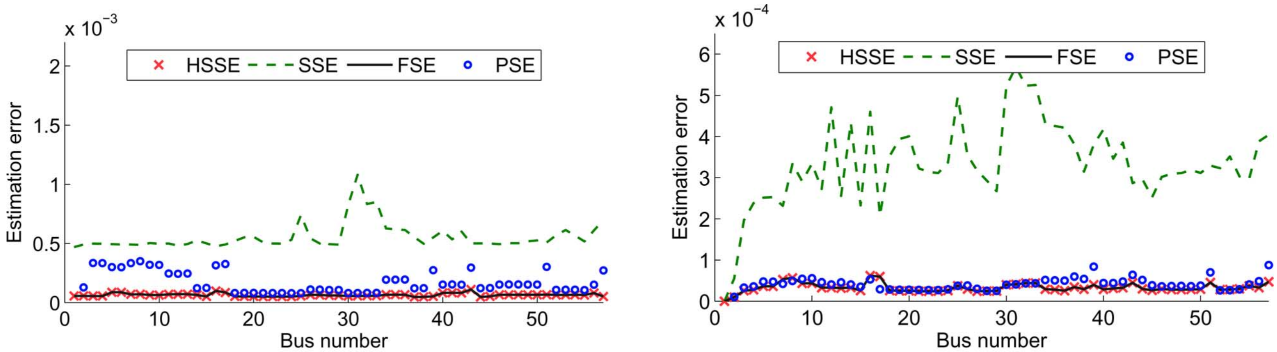
In those cases, the multistage estimation architecture based on the fusion principles enables one to take full advantage of the extra redundancy and enhanced accuracy provided by phasor measurements. In fact, depending on the location of the PMUs in the power network, estimation errors due to bad data on critical measurements and critical sets can be prevented through the use of the fusion strategy, as demonstrated in the preliminary studies on this topic reported in [31].

The problem which remains to be addressed is the occurrence of bad data among phasor measurements. Simply replicating the above described SSE strategy at the PSE stage would imply PMU-observability. Since this assumption is seen as unrealistic at least for some years to come, alternative approaches must be considered.

We thus envision a strategy which relies on making both SCADA and PMU measurement values available to be processed once again, this time at a stage following the computation of the final fused estimates. Such processing would take place either as a final step within the fusion stage itself, or at an extra real-time modeling module (a final "result validation" stage). In either case, by using 1) the final estimation fusion results; 2) measurement values previously employed to generate both $\hat{\mathbf{x}}_S$ and $\hat{\mathbf{x}}_P$, and 3) the corresponding error covariance matrices, then residual or estimation error analysis can be performed to investigate the presence of bad data among phasor measurements. Furthermore, gross SCADA measurements that could have passed unnoticed in the SSE stage could also be identified at this phase. This rationale will be explored in future research efforts on this topic.

VI. Simulation Results

This section illustrates the application of the proposed multi-stage Fusion State Estimator, FSE, through several case studies conducted with the IEEE 57-bus, 118-bus, and 300-bus test systems. Data for these networks are available in [32]. Three distinct estimators are employed as references to evaluate the FSE performance: a hybrid simultaneous estimator (HSSE), a conventional SCADA-based estimator (SSE) and an estimator that only processes phasor measurements (PSE). All estimators make use of an orthogonal, Givens rotations-based WLS solver, which also exhibits *a priori* information processing capability [4]. They have been implemented in FORTRAN, using the Intel Visual Fortran compiler, and run on a 2.20 GHz, 6.0 GB of RAM, Intel Core i7 computer. A Gaussian random number generator is used to simulate the measurement errors, whose assumed accuracy levels are 1% for SCADA and 0, 1% for PMU measurements. Reported results for each case are obtained by averaging the outcomes of one hundred performed simulations, each of which considering distinct measurement errors. The SCADA metering schemes are composed of active/reactive power injection, active/reactive power flow, and bus voltage magnitude measurements, which are evenly distributed throughout the network. All power measurements are taken in active/reactive pairs. PMU metering schemes comprise bus voltage and branch current phasor measurements. The *a*



(a) (b)

Fig. 3. Absolute value of estimation errors. Case A, IEEE 57-bus system. (a) Voltage magnitude error. (b) Voltage phase angle error.

TABLE I
NUMBER OF SCADA AND PMU MEASUREMENTS

	Case A					Case B				
	P/Q	$\|V\|$	t/u	$\dot{V}$	$\dot{I}$	P/Q	$\|V\|$	t/u	$\dot{V}$	$\dot{I}$
IEEE 57-bus	37	36	48	46	41	37	36	48	11	4
IEEE 118-bus	72	67	116	86	94	72	67	116	35	31
IEEE 300-bus	-	-	-	-	-	234	222	305	145	175

TABLE II
TOTAL NUMBER OF MEASUREMENTS FOR EACH SYSTEM

	Case A			Case B		
	m_{SSE}	m_{PSE}	$n_{\bar{x}}$	m_{SSE}	m_{PSE}	$n_{\bar{x}}$
IEEE 57-bus	206	87	0	206	15	84
IEEE 118-bus	443	180	0	443	66	112
IEEE 300-bus	-	-	-	1300	320	134

priori state values used in Section VI-B are equal to $1.0pu\angle 0^\circ$, with variances in the 1×10^8 range.

To assess the performance of the estimators, we make use of the mean of estimation errors and the voltage metric defined in [33]

$$Macc_{\text{v}} = \left\|\dot{V}^{\text{error}}\right\|_2 = \left(\sum_j \left|\dot{V}_j^{\text{true}} - \dot{V}_j^{\text{est}}\right|^2\right)^{\frac{1}{2}} \tag{19}$$

where $\dot{V}_j^{\text{true}}$ and $\dot{V}_j^{\text{est}}$ are the "true" and estimated complex phasor voltage at the jth bus, respectively.

Simulations are grouped into three cases, referred to as cases A, B and C, which are characterized as follows:

- **Case A**—The base case, in which the electrical network is both SCADA- and PMU-observable.
- **Case B**—It is assumed that the electrical network is SCADA-observable but the number of deployed PMUs is insufficient to ensure PMU-observability. *A priori* state values equal to $1.0\ pu\angle 0^\circ$ are then assigned to the unobservable buses in order to restore observability (see Section V-C).
- **Case C**—As in Case B, *a priori* state information is employed to artificially ensure observability. However, the *a priori* state values are now based on the results of a previous state estimation study. In this case, simulations are conducted only on the 57-bus network.

The metering schemes used in Cases A and B are summarized in Tables I and II. For Case C, the same metering scheme of Case B is employed. The notation used in Table I is as follows: P(Q) stands for active (reactive) power injection measurements; t(u) refers to active (reactive) power flow measurements; and |V| stands for voltage magnitude measurements. All of the above are SCADA measurements. $\dot{V}(\dot{I})$ represents voltage (current) phasor measurements. The total number of measurements for each test system and each case, as well as the amount of *a priori* data required to ensure observability at the PSE module are presented in Table II. In this table, m_{SSE} and m_{PSE} stand for the number of SCADA and phasor measurements, respectively, and $n_{\bar{x}}$ denotes the number of state variables unobservable with only phasor measurements (and therefore the amount of required *a priori* state data). It is assumed that each PMU provides a bus voltage phasor measurement, and may also supply one or more current phasor measurements on branches incident to the same bus.

A. SCADA- and PMU-Observable Network

Simulations in this section consider the ideal scenario in which the PMU penetration is sufficient to ensure network observability. Fig. 3 presents the estimation errors associated to the results provided by all simulated strategies, for both voltage magnitude and phase angle at all buses of the 57-bus system. The FSE plot clearly shows that the results of the conventional SCADA estimator, labeled as SSE, are greatly enhanced through their fusion with the PSE estimates. In addition, it is also visible the coincidence of the FSE trace with the one produced by the optimal HSSE estimator. The results obtained for the 118-bus system are presented in Fig. 4 and basically follow the same patterns already observed in Fig. 3. We can thus conclude that the simulations for both test systems confirm the optimality of the FSE results under full PMU-observability conditions, as discussed in Section IV-C.

The numerical performance indices computed for all estimators shown in Table III are fully consistent with the plots in Figs. 3 and 4 and also confirm the FSE theoretical properties. Notice that the numerical values of all indices calculated for the two networks indicate the same performance of the FSE and the HSSE strategies. Furthermore, a reduction of about 90% in the voltage metric is achieved by using the FSE in lieu of the SSE approach, which quantifies the quality gains yielded by the estimation fusion strategy in this case. Finally, Table IV

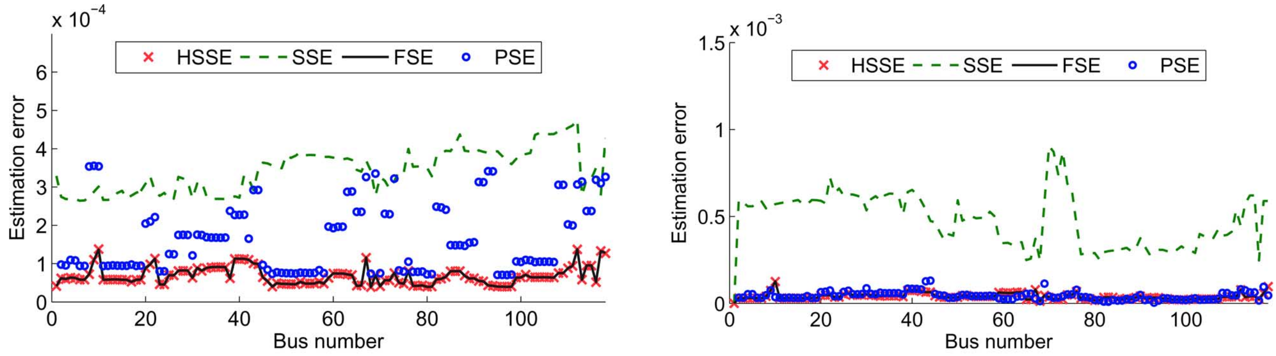


Fig. 4. Absolute value of estimation errors. Case A, IEEE 118-bus system. (a) Voltage magnitude error. (b) Voltage phase angle error.

TABLE III
PERFORMANCE INDEXES FOR CASE A

(all results as factors of 10^{-3})

	IEEE 57-bus			IEEE 118-bus		
Estimator	Voltage Metric	Error Mean $\|V\|$	δ	Voltage Metric	Error Mean $\|V\|$	δ
SSE	8.30	0.37	0.37	7.70	0.34	0.48
PSE	1.80	0.17	0.04	2.60	0.17	0.04
HSSE	0.67	0.03	0.03	1.10	0.07	0.04
FSE	0.68	0.03	0.03	1.10	0.07	0.04

TABLE IV
DEGREE OF SUBOPTIMALITY FOR CASE A

	IEEE 57-bus		IEEE 118-bus	
Estimator	$J(\hat{\mathbf{x}})$	μ_{SubOpt}	$J(\hat{\mathbf{x}})$	μ_{SubOpt}
HSSE	7.049	0.033 %	22.401	0.049 %
FSE	7.051		22.412	

compares the WLS objective function values obtained from the FSE and HSSE approaches, and also presents the degree of suboptimality calculated for the two benchmark systems. Considering the magnitude of $J(\hat{\mathbf{x}})$ values, the difference in objective function values are in the numerical noise range, so that the degrees of suboptimality computed for both systems are virtually zero. This once more confirms the theoretical conclusions of Section IV-C.

B. Unobservable Network With Respect to Phasor Measurements: Flat Voltage Profile as A Priori Information

This subsection considers the realistic situation in which the PMU deployment is limited, so that the network is not PMU-observable. As remarked in Section V-C, under such conditions the use of *a priori* information is essential to get a solution from the PSE module.

Table V presents the values of the performance indices for all considered strategies and all three test systems. It becomes immediately clear that PSE results are not useful by themselves, what is expected due to the use of arbitrary *a priori* state values to restore PMU-observability. However, even under those severe conditions, the FSE and HSSE indices in Table V indicate that those estimators outperform the conventional SSE strategy. This conclusion is confirmed by the estimated state errors provided in Figs. 5 and 6 for the 57- and 118-bus networks, respectively (results for the 300-bus system are omitted due to space limitations). It is again possible to see that the proposed fusion estimator is able to take advantage of the PSE estimates, even when obtained from a limited number of PMU data, in order to enhance the conventional SSE results. Also, FSE estimation errors are consistently close to, actually almost coincident with, those of the optimal HSSE strategy. As discussed at Section II-B, due to the high variance values ascribed to *a priori* information, the fusion procedure takes care of "filtering out" the influence of the arbitrary state values, so that they do not affect the FSE results.

The arbitrary *a priori* information needed to ensure observability has some effect on the optimality of the fusion solution, as antecipated in Section V-C. This can be observed by comparing the degrees of suboptimality in Table VI with those of Section VI-A, shown in Table IV. Although the performance degradation with respect to the optimal strategy in the current case is more noticeable, it still remains in a very low range, indicating that the quality of the FSE results deviates very little from the optimal performance.

C. Unobservable Network With Respect to Phasor Measurements: Previous Estimate as A Priori Information

This case differs from Case B in that, in order to circumvent the unobservability of the network with respect to the available set of phasor measurements, complex voltage estimates obtained from a previous state estimation run are used in lieu of a flat voltage profile at the PSE module. Only the results for the 57-bus network are presented for this case.

The situation conceived in this experiment assumes that the previous estimation results corresponds to the system operating condition which immediately precedes the one of interest. It is also assumed that system loading is increasing, and the total load at the previous operating point is 98% of the current condition. Such a reducing factor is applied to all individual loads of the network, and the complex bus voltages computed for this load condition are used as *a priori* state information at the unobservable buses. The variances assigned to that *a priori* data are in the range of 1×10^{-1}, that is, they are respectively one and two degrees of magnitude larger than that of SCADA and phasor measurements, although significantly less than the value used in Section VI-B for *a priori* information.

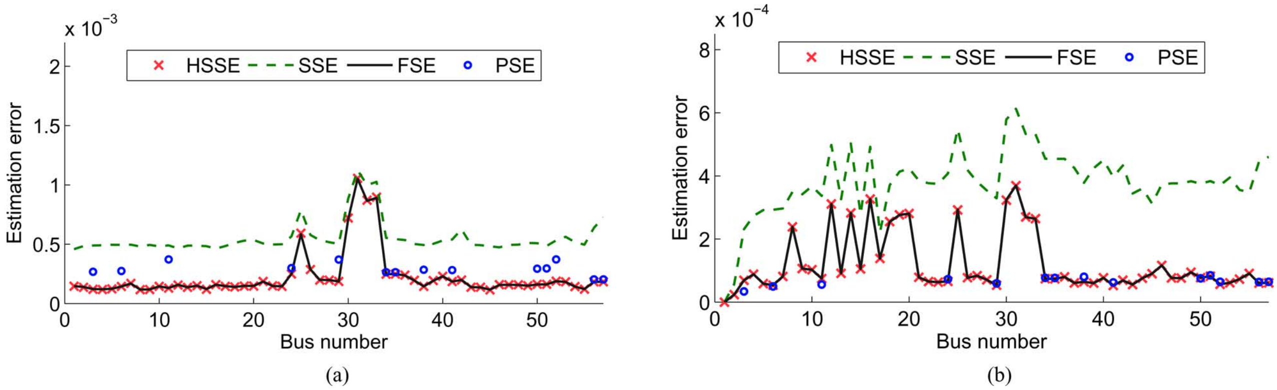


Fig. 5. Absolute value of estimation errors. Case B, IEEE 57-bus system. (a) Voltage magnitude error. (b) Voltage phase angle error.

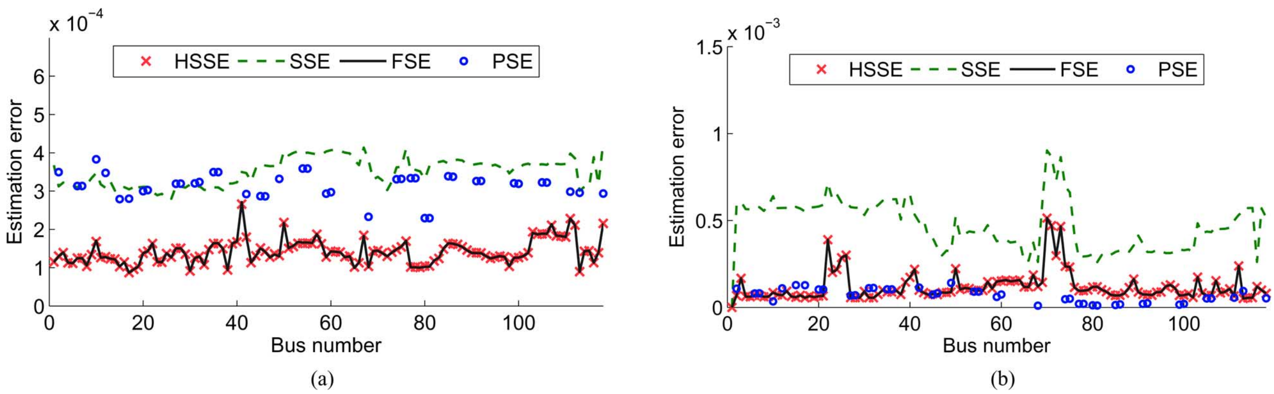


Fig. 6. Absolute value of estimation errors. Case B, IEEE 118-bus system. (a) Voltage magnitude error. (b) Voltage phase angle error.

TABLE V
PERFORMANCE INDEXES FOR CASE B

	IEEE 57-bus			IEEE 118-bus			IEEE 300-bus		
	Voltage	Error Mean		Voltage	Error Mean		Voltage	Error Mean	
Estimator	Metric	$\|V\|$	δ	Metric	$\|V\|$	δ	Metric	$\|V\|$	δ
SSE	7.9×10^{-3}	6.5×10^{-4}	3.6×10^{-4}	7.6×10^{-3}	3.5×10^{-4}	3.7×10^{-4}	2.2×10^{-2}	3.7×10^{-4}	8.9×10^{-4}
PSE	1.5×10^{0}	5.3×10^{-2}	1.6×10^{-1}	1.8×10^{0}	1.4×10^{-2}	1.1×10^{-1}	2.7×10^{0}	9.6×10^{-3}	6.5×10^{-2}
HSSE	6.0×10^{-3}	3.3×10^{-4}	1.4×10^{-4}	2.6×10^{-3}	1.4×10^{-4}	1.8×10^{-4}	5.1×10^{-3}	1.1×10^{-4}	7.5×10^{-5}
FSE	6.0×10^{-3}	3.3×10^{-4}	1.4×10^{-4}	2.6×10^{-3}	1.4×10^{-4}	1.8×10^{-4}	5.1×10^{-3}	1.1×10^{-4}	7.5×10^{-5}

TABLE VI
DEGREE OF SUBOPTIMALITY FOR CASE B

	IEEE 57-bus		IEEE 118-bus		IEEE 300-bus	
Estimator	$J(\hat{\mathbf{x}})$	μ_{SubOpt}	$J(\hat{\mathbf{x}})$	μ_{SubOpt}	$J(\hat{\mathbf{x}})$	μ_{SubOpt}
HSSE	3.162	0.167%	22.401	0.263%	74.660	0.161%
FSE	3.167		22.412		74.781	

TABLE VII
PERFORMANCE INDEXES FOR CASE C

	IEEE 57-bus		
	Voltage	Error Mean	
Estimator	Metric	$\|V\|$	δ
SSE	7.9×10^{-3}	6.5×10^{-4}	3.6×10^{-4}
PSE	8.2×10^{-2}	1.3×10^{-3}	9.7×10^{-3}
FSE	6.0×10^{-3}	3.3×10^{-4}	1.4×10^{-4}

The results obtained with the new definition of *a priori* state values are summarized in Table VII. By comparing them with those of Case B presented in Table V, we immediately see that the metrics associated to the performance of the PSE module take now significantly lower values. This indicates that the results provided by PSE for the PMU-unobservable states become much more accurate. However, due to the difference of variances (and consequently of weights) with respect to the SCADA data, the results of the SSE module again prevail at the fusion stage, so that the FSE results for cases B and C are practically the same, as one can conclude by comparing Tables V and VII.

D. Assessment of Computational Effort

This subsection provides a preliminary quantitative appraisal of the estimation fusion strategy computational performance. The experiments conducted for that purpose are based on some premises, as follows:

1) All estimators involved in this assessment make use of the same state estimation software, whose WLS solver is based on a Givens rotations algorithm capable of processing *a priori* state values [20]. Sparsity techniques are employed

TABLE VIII
COMPUTING TIME (% OF t_{HSSE})

	IEEE 57-bus		**IEEE 118-bus**		**IEEE 300-bus**
	Case A	Case B	Case A	Case B	Case B
t_{HSSE}	100.0	100.0	100.0	100.0	100.0
t_{SSE}	19.3	55.4	25.2	42.4	29.8
t_{PSE}	8.8	7.0	3.2	2.5	0.4
t_{FUSION}	13.7	37.0	7.1	10.3	1.0
t_{FSE}	33.0	92.4	32.4	52.7	30.9

in the implementation, but a "natural ordering" scheme induced by bus numbering is used, instead of a more sophisticated scheme aimed at reducing fill-ins caused by the rotations.

2) The implementation of the fusion stage is based on the sparse solution of (17), derived in Section V-B of this paper.
3) For a given test system, the computing times for all FSE stages are expressed as a percentage of the execution time required by the conventional hybrid simultaneous estimator described in Section III-A. In other words, in each case study the HSSE execution time is taken as the base to compute the relative computing times for the other estimators.
4) Since the SCADA- and PMU-based estimators constitute separate modules and process distinct measurement sets (see Fig. 2), it is reasonable to consider a parallelization of the SSE and PSE processes. Accordingly, if t_{SSE}, t_{PSE} and t_{FUSION} denote SSE, PSE and fusion stage execution times, respectively, then the computing time for the whole estimation fusion process, t_{FSE}, is given by

$$t_{\text{FSE}} = \max(t_{\text{SSE}}, t_{\text{PSE}}) + t_{\text{FUSION}}. \tag{20}$$

Table VIII presents the computing times for each individual stage, as well as the time requirements of the whole estimation fusion process. The following remarks apply:

- For both 57-bus and 118-bus systems, t_{SSE} as a percentage of t_{HSSE} varies from case A to case B, despite the fact that precisely the same measurement sets are processed by the SSE estimator in both cases (as a matter of fact, the respective absolute times in seconds are equal). The difference between the relative values happens because t_{HSSE} is significantly larger in case A than in case B for both networks, due to the larger number of phasor measurements processed in the former case. The same effect explains the difference in the corresponding values of t_{FUSION}.
- In all cases, $t_{\text{PSE}} \ll t_{\text{SSE}}$, what is expected due to the characteristics of the distinct measurement sets processed by the SSE and PSE estimators. In addition to dealing with a larger data set, SSE processes power injection measurements, which give rise to more nonzero entries in the Jacobian matrix rows than phasor measurements.
- Looking now across all test systems and focusing attention on case B, which is the most realistic considering the current stage of PMU penetration in power networks, results in Table VIII show that relative t_{FSE} values decrease with the size of the network. Although factors such as nonuniform $m_{\text{PSE}}/m_{\text{SSE}}$ ratios for the three systems, use of efficient ordering schemes, etc., should be more carefully evaluated, the results seem to indicate that the computational performance of the proposed architecture *as compared with the HSSE architecture* tends to improve as system size increases.

TABLE IX
COMPUTING TIME FOR THE FSE STRATEGY (S)

IEEE 57-bus		**IEEE 118-bus**		**IEEE 300-bus**
Case A	Case B	Case A	Case B	Case B
0.056	0.046	0.235	0.230	4.097

Finally, Table IX provides the absolute execution times in seconds required by the FSE strategy, for all previous cases and all test systems. Although there is still room for many enhancements regarding the implementation of the various estimation stages, those values indicate that the proposed estimation architecture is able to provide computing times compatible with real time application.

VII. CONCLUSION

This paper introduces a multistage state estimator based on Multisensor Data Fusion theory to optimally combine results independently obtained from SCADA- and PMU-based estimation modules. The modular architecture of the proposed estimator accommodates the distinct characteristics of both technologies, and does not impose internal changes to the structure of existing EMS software. In addition, any valid PSSE algorithm is elegible to be used as the basis for the SCADA and PMU estimation modules.

Previously developed theoretical principles and methods encountered in the rich literature on Multisensor Data Fusion are reviewed in detail before being applied to the PSSE problem. Particular attention is then given to issues such as optimality, computational aspects and observability with respect to phasor measurements. The paper provides both theoretical arguments and simulation results to support the conclusion that, under full SCADA- and PMU-observability conditions and assuming uncorrelation between both classes of measurements, the Fusion State Estimator provides the same optimal results of a hybrid simultaneous state estimator. Even when PMU-observability cannot be ensured, and provided that the accuracy of phasor measurements is superior as compared to conventional measurements, the fusion approach still provides better results than conventional SCADA-based estimators. Extensive simulations conducted with the IEEE 57-bus, 118-bus, and 300-bus benchmark systems are used to illustrate the benefits of the proposed estimation architecture.

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