Presented at the 2012 PMAPS Conference, Istambul, Turkey, 10-14 June 2012.

Enhanced Probabilistic Modeling of Phasor Measurement Errors in Hybrid SCADA-PMU State Estimation

Danie Bez, *Student Member, IEEE,* Antonio Simões Costa, *Fellow, IEEE*

***Abstract*—This paper addresses the probabilistic modeling of phasor measurement errors in power system state estimation. Phasor measurement units (PMUs) provide the magnitude and phase angle of monitored synchrophasors. However, computational efficiency prescribes their conversion to rectangular components prior to processing by a weighted least-squares state estimator. Since conventional estimators treat data in a scalar basis, the correlation between measurement components has to be neglected, thus affecting the statistical properties of the weighted least-squares solution and the quality of the final estimates. To circumvent that drawback, a blocking scheme applied in connection with orthogonal estimators is proposed to allow the simultaneous processing of phasor measurements' real and imaginary parts. As a consequence, correlations between the rectangular components of measurement errors, as well as their influence on the final estimates, can be fully taken into account.**

The paper describes the principles and implementation of the blocked orthogonal solution and its insertion into an architecture for combining SCADA and PMU estimates. Simulation results based on *IEEE* benchmarks systems are used to illustrate the proposed approach.

***Index Terms*—Probabilistic modeling of measurement errors, synchronized phasor measurements, Power system state estimation, Givens rotations.**

I. Introduction

Power Systems State Estimation (PSSE) is the basic tool for power system independent operators. The state estimator processes redundant noisy measurements, is able to efficiently detect bad data and returns a snapshot of the system operating point. Traditionally, measurements processed by the state estimator are obtained from the supervisory control and data acquisition (SCADA) system, which scans remote terminal units (RTU) located at the substation level. Recently, the advent of the synchronized phasor measurement technology has made it possible to directly measure electric current and voltage phasors, something previously unattainable with SCADA. Phasor and SCADA measurements exhibit some distinct characteristics [1]. While SCADA measurement scans take place at about every 5 seconds, PMU sampling rates are in the range of tenths of samples per second. In addition, the latter tend to be more accurate. When placed at strategic points of the network, PMUs are also able improve observability [2], as well as to enhance bad data detection [3].

Among the available alternatives to incorporate phasor measurements into the state estimation process, a particularly attractive one from a practical perspective consists in adding an additional estimation module to the SCADA-based estimator, so that the distinct types of measurements are processed by distinct estimation stages [4]–[6]. The first stage comprises a conventional state estimator, which processes SCADA measurements only. The second stage receives the estimates provided by the first stage and uses them as *a priori* state information for the processing of the PMU measurements. This approach presents the advantage of maintaining unchanged all the structure already in use by power operators/utilities. Another benefit is that the estimator in the second stage can be made linear, since it processes only complex voltages and currents whose relationships with the state variables are linear when rectangular coordinates are used [4].

However, previous attempts to implement such estimation architecture require strong approximations concerning the statistical properties of the measurement errors, namely, errors on the real and imaginary parts of a given quantity have to be assumed as mutually uncorrelated. This affects the statistical properties of the weighted least squares solution, as well as the quality of the final estimates.

This paper proposes an enhanced alternative to incorporating PMU measurements into the post-processing stage which circumvents the need of adopting the abovementioned uncorrelatedness assumption. The proposed approach takes advantage of the linear measurement model provided by formulating the problem in rectangular coordinates, and makes use of a generalized blocked form of Givens orthogonal rotations which has been especially developed for this particular application. The blocked form of the orthogonal rotations make it possible to jointly process the real and imaginary parts of measured quantities, so that the correlation between them can be fully taken into account. It also exhibits the same property in regard to the *a priori* information imported from the first estimation stage, which can thus be also considered correlated and, in addition, is processed with no extra computation cost by the orthogonal estimator. Therefore, the proposed approach can be seen as a step forward in terms of preserving the statistical properties of the estimation problem.

This paper is organized as follows. Section II reviews the basic principles of PSSE. Its solution through scalar Givens rotations is revisited in Section III, where their generalization towards the blocked form is also introduced. Section IV shows how the blocked rotations are embedded into the proposed hybrid state estimation architecture. Results of numerical simulations are presented in Section V, which is then followed by the concluding remarks.

II. State Estimation Background

A. Conventional State Estimation

Conventional state estimation is based on SCADA measurements: voltage and current magnitudes, active and reactive injections, active and reactive power flows. Considering an *n* bus system with *m* measurements, the measurement model is given by

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \eta \tag{1}$$

where $\mathbf{z}$ is the $m \times 1$ measurements vector, $\mathbf{x}$ is the $n \times 1$ power system state vector, $\mathbf{h}$ is the $m \times 1$ set of nonlinear functions relating the measurement to the states and η is the $m \times 1$ vector of random measurement errors, whose *i-th* component has variance equal to σ_i^2. The measurement error vector is assumed as Gaussian distributed with zero mean, that is,

$$E\{\eta\} = 0 \tag{2}$$

Errors in SCADA measurements are usually assumed uncorrelated, which implies that their covariance matrix $\mathbf{R}$ is reduced to its diagonal entries. Those are the measurement variances σ_i^2, obtained from the accuracy properties of the metering devices. Thus,

$$E\{\eta\eta^t\} = \mathbf{R} = diag\{\sigma_1^2, ..., \sigma_m^2\} \tag{3}$$

The state estimation problem is usually formulated as a weighted least squares (WLS) problem, so that the weighted sum of squared residuals is minimized. The objective function of the WLS problem is given by

$$Min J(\hat{\mathbf{x}}) = \frac{1}{2}(\mathbf{z} - \mathbf{h}(\hat{\mathbf{x}}))^t \mathbf{R^{-1}} (\mathbf{z} - \mathbf{h}(\hat{\mathbf{x}})) \tag{4}$$

The conventional solution of this optimization problem is iteratively obtained through the Gauss-Newton method, starting at a given point $\mathbf{x}^k$. Incremental corrections to $\mathbf{x}^k$ are given by the solution of the *Normal Equation* [7]–[9]:

$$(\mathbf{H}^t\mathbf{R}^{-1}\mathbf{H})\Delta\mathbf{x} = \mathbf{H}^t\mathbf{R}^{-1}\Delta\mathbf{z} \tag{5}$$

where $\mathbf{H}$ is the Jacobian matrix of of $\mathbf{h}(\mathbf{x})$, computed at $\mathbf{x}^k$ and $\Delta\mathbf{z} = \mathbf{z} - \mathbf{h}(\mathbf{x^k})$. Convergence is reached when $\Delta\mathbf{x}$ becomes smaller than a given tolerance. Assuming that $\hat{\mathbf{x}}$ is the final state estimate, the covariance matrix of the estimation erros is given by

$$Cov(\hat{\mathbf{x}} - \mathbf{x}) = \mathbf{C_x} = (\mathbf{H}^t\mathbf{R}^{-1}\mathbf{H})^{-1} \tag{6}$$

B. A priori State Estimation

Whenever some prior knowledge is available about the state variables, it can be embedded in the estimation process as *a priori* information. This can be accomplished by adding a new term to the objective function, given by

$$\frac{1}{2}(\hat{\mathbf{x}} - \bar{\mathbf{x}})^t\mathbf{P}^{-1}(\hat{\mathbf{x}} - \bar{\mathbf{x}}) \tag{7}$$

where $\bar{\mathbf{x}}$ is the $n \times 1$ random vector of *a priori* information on the state variables and $\mathbf{P}$ is its covariance matrix. If the elements errors of $\bar{\mathbf{x}}$ are assumed uncorrelated, then $\mathbf{P} = diag\{\bar{\sigma}_1^2, ..., \bar{\sigma}_n^2\}$, where $\bar{\sigma}_i^2$ is the variance representing the uncertainty on the value $\bar{\mathbf{x}}_i$. The minimization of the augmented objective function obtained by taking *a priori* state information into account leads to the extended normal equation:

$$(\mathbf{H}^t\mathbf{R}^{-1}\mathbf{H} + \mathbf{P}^{-1})\Delta\mathbf{x} = \mathbf{H}^t\mathbf{R}^{-1}\Delta\mathbf{z} + \mathbf{P}^{-1}\Delta\bar{\mathbf{x}} \tag{8}$$

where $\Delta\bar{\mathbf{x}} = \bar{\mathbf{x}} - \mathbf{x}^k$. When *a priori* state information is considered, the problem is referred to as *A priori* State Information *(APSI)* State Estimation. As shown in the sequel, *a priori* information can be accommodated within the weighted least-squares framework at no extra computational cost when the problem is solved through the three-multiplier (3M) version of orthogonal Givens rotations.

III. State Estimation via 3M Givens rotations

Orthogonal techniques to solve the least square problem have been introduced in the past to improve the numerical robustness of least-squares problems. This section describes a sequential orthogonal method based on the three-multiplier version of Givens rotations, first applied to state estimation in [10].

A. Scalar form of Givens rotations

Consider the linearized version of the least-squares problem (4) with measurement model $\mathbf{z} = \mathbf{H}\mathbf{x} + \eta$. Assume that an initial measurement vector $\mathbf{z}_0$ is selected which has the same size of the state vector, so that the corresponding observation matrix $\mathbf{H_0}$ is square. In addition, assume also that a new measurement z_1, related to the state variables as $z_1 = \mathbf{h}_1^t\mathbf{x} + \eta_1$, is to be processed. The rows of $\mathbf{H}_0$ and $\mathbf{h}_1^t$ are scaled by factors $\mathbf{R}_0^{-1/2}$ and $w^{1/2}$, respectively. Then a sequence of plane (Givens) rotations $\mathbf{Q}$ can be applied to the scaled rows of the new observation matrix (augmented with the corresponding measurements) so that [10]:

$$\mathbf{Q}\left(\begin{bmatrix} \mathbf{R}_0^{-\frac{1}{2}} & \\ & w^{\frac{1}{2}} \end{bmatrix}\left[\begin{array}{c|c} \mathbf{H}_0 & \mathbf{z}_0 \\ \mathbf{h}_1^t & z_1 \end{array}\right]\right) = \left[\begin{array}{c|c} \mathbf{U} & \mathbf{c} \\ \mathbf{0} & e \end{array}\right] \tag{9}$$

where $\mathbf{U}$ is a $n \times n$ upper triangular matrix, $\mathbf{c}$ is a $n \times 1$ vector, $\mathbf{0}$ is a $1 \times n$ null vector and e is an scalar. Assuming observability [8], the estimated state vector $\hat{\mathbf{x}}$ based on the processed measurements can be obtained by simply solving the following triangular system of equations:

$$\mathbf{U}\ \hat{\mathbf{x}} = \mathbf{c} \tag{10}$$

Furthermore, the weighted sum of the squares residuals is a by-product of the rotations, and is determined from e. Since matrix $\mathbf{Q}$ which stores the sequence of rotations is orthogonal, this method to solve WLS problems is superior to the conventional normal equation approach in terms of numerical robustness [10].

B. Three Multiplier (3M) scalar Givens rotations

The three-multiplier (3M) version of Givens rotations is based on the factorization of matrix $\mathbf{U}$ as [11], [12]

$$\mathbf{U} = \mathbf{D}^{\frac{1}{2}}\bar{\mathbf{U}} \tag{11}$$

where $\mathbf{D}$ is diagonal and $\bar{\mathbf{U}}$ is a *unit* upper triangular matrix. The artifice of scaling $\mathbf{U}$ makes this scheme computationally superior to the original Givens method, because it eliminates square roots calculations implicit in the original rotations (in practice $\mathbf{D}^{\frac{1}{2}}$ is not required, only $\mathbf{D}$ is actually calculated). Three-multiplier Givens rotations are the basis for a recursive method which processes each row of the Jacobian matrix (augmented by the corresponding entry of $\boldsymbol{\Delta}\mathbf{z}$) at a time. It also makes it possible to add *a priori* information and to obtain the weighted sum of squared residuals with no extra computational cost.

To illustrate the *3M* Givens rotations, consider that a new row vector $\mathbf{p}$, which corresponds to a row of $[\mathbf{H} \mid \boldsymbol{\Delta}\mathbf{z}]$, undergoes rotations with a row vector $\mathbf{u}$ from the matrix $\mathbf{U}$:

$$\begin{array}{l} \mathbf{u} = [\ 0 \ \cdots \ 0 \ \ \sqrt{d} \ \ \cdots \ \sqrt{d}\bar{u}_k \ \cdots \ \sqrt{d}\bar{u}_{n+1}\ ] \\ \mathbf{p} = [\ 0 \ \cdots \ 0 \ \sqrt{w}p_i \ \cdots \ \sqrt{w}p_k \ \cdots \ \sqrt{w}p_{n+1}\ ] \end{array} \tag{12}$$

Both vectors have already been scaled according to (11), the scaling factors being $\sqrt{d}$ and $\sqrt{w}$, respectively. After a single rotation, the $i-th$ of $\mathbf{p}$ is zeroed out and the row vectors take the form

$$\begin{array}{l} \mathbf{u}' = [\begin{array}{ccccccc} 0 & \cdots & 0 & \sqrt{d'} & \cdots & \sqrt{d'}\bar{u}'_k & \cdots & \sqrt{d'}\bar{u}'_{n+1} \end{array}] \\ \mathbf{p}' = [\begin{array}{ccccccc} 0 & \cdots & 0 & 0 & \cdots & \sqrt{w'}p'_k & \cdots & \sqrt{w'}p_{n+1} \end{array}] \end{array} \quad (13)$$

Next, elementary rotations are sequentially performed in order to annihilate all non-zero element of $\mathbf{p}$. This process introduces changes in $\mathbf{U}$, $\mathbf{c}$ and e after each rotation.

As far as PSSE is concerned, one the most important features of the scaling mechanism is that it allows the weighting of each new measurement with no extra calculation. In fact, proper weighting of measurement Δz_i is achieved when factor w_i is given by

$$w_i = \frac{1}{\sigma_i^2} \quad (14)$$

where σ_i^2 is the variance of the measurement z_i.

In regard to factor d_i, its value at the initialization of the rotation process can be seen as the initial weight for state variable *i* before any measurement is processed. In other words, $d_i^{(0)}$ corresponds to the weighting factor of the *a priori* information available about the states. Therefore

$$d_i^{(0)} = \frac{1}{\bar{\sigma}_i^2} \quad (15)$$

where $\bar{\sigma}_j^2$ is the variance of the *a priori* information on the state variable *j*.

If no *a priori* information is available about the states, then $d_i^{(0)} = 0$. From (11), this implies that $\mathbf{U}$ is initially a null triangular matrix. If, on the contrary, there is prior information on the states, the 3M Givens rotations scheme is initialized as follows:

- d_i as in equation (15);
- vector $\mathbf{c}$ as the available *a priori* information, that is,

$$\mathbf{c} = \bar{\mathbf{x}}_{priori}. \quad (16)$$

To summarize, the 3M Givens rotations method can easily consider *a priori* information in the very initialization stage of the estimation process, so that no extra computational cost is incurred.

C. Block form of Givens rotations

The 3M form of Givens rotations described in the previous section considers a *scalar* weighting scheme in which the weight assigned to each measurement depends solely on its variance. This implicitly means that possible cross-correlations between measurements must be neglected. Although acceptable in certain applications, such as conventional SCADA-based state estimation, such assumption implies a strong statistical simplification for others. Such is the case when voltage and current phasor measurements converted to rectangular form are to be processed by the state estimator, since the real and imaginary parts of such phasors tend to be strongly correlated.

To circumvent that limitation, an extended *blocked form* of the 3M Givens rotations has been developed in connection with this research work in order to consider the covariance between two distinct measurements. The proposed extension enables the processing of two measurements at a time, which can thus be seen as statistically coupled together, since their weighting factor becomes the inverse of the 2×2 covariance matrix of the paired observations. To outline the procedure, consider that measurements are now organized in pairs and the errors for each pair are assumed *correlated*. This implies that the covariance matrix $\mathbf{R}$ is *block diagonal*, that is,

$$E(\eta\eta^t) = \tilde{\mathbf{R}} = \begin{bmatrix} \sigma_1^2 & c_{12} & & & & \\ c_{21} & \sigma_2^2 & & & & \\ & & \sigma_3^2 & c_{34} & & \\ & & c_{43} & \sigma_4^2 & & \\ & & & & \ddots & \ddots \\ & & & & \ddots & \ddots \end{bmatrix} \quad (17)$$

where σ_i^2 is the variance of measurement i of the pair and $c_{ij} = c_{ji}$ stands for the covariance between the pair components.

Equation (9), which prescribes how new measurements are processed by the 3M Givens rotations, is then generalized. Accordingly, vector $\mathbf{z}_1$ is now composed by a 2×1 pair of new measurements, which are related to the states by the $2 \times n$ matrix $\mathbf{H}_1$. The proposed blocked form of the rotations is then applied to matrix $[\mathbf{H_0}^t \mid \mathbf{H}_1^t]^t$ augmented by vector $[\mathbf{z_0}^t \mid \mathbf{z}_1^t]^t$ (both previously scaled by $\mathbf{R}^{-\frac{1}{2}}$ and $\mathbf{W}^{\frac{1}{2}}$) in order to obtain an upper triangular linear system of equations. If $\tilde{\mathbf{Q}}$ represents the matrix that stores the rotations in the new blocked form, we have:

$$\tilde{\mathbf{Q}} \left(\begin{bmatrix} \mathbf{R}_0^{-\frac{1}{2}} & \\ & \mathbf{W}^{\frac{1}{2}} \end{bmatrix} \left[\begin{array}{c|c} \mathbf{H}_0 & \mathbf{z}_0 \\ \mathbf{H}_1 & \mathbf{z}_1 \end{array} \right] \right) = \left[\begin{array}{c|c} \tilde{\mathbf{U}} & \tilde{\mathbf{c}} \\ \mathbf{0} & \tilde{\mathbf{e}} \end{array} \right] \quad (18)$$

where $\tilde{\mathbf{c}}$ is a $n \times 2$ vector, $\mathbf{0}$ is a $2 \times n$ null matrix, $\tilde{e}$ is a 2×2 matrix and $\tilde{\mathbf{U}}$ is a $n \times n$ upper triangular matrix with 2×2 block identities on its main diagonal.

The estimated state vector $\hat{\mathbf{x}}$ is obtain by solving the triangular system of equations by back substitution.

$$\tilde{\mathbf{U}}\ \hat{\mathbf{x}} = \tilde{\mathbf{c}}_1 \quad (19)$$

where $\tilde{\mathbf{c}}_1$ is the first column of $\tilde{\mathbf{c}}$.

The block form of the 3M Givens rotations is based on the same factorization as in (11), but now $\mathbf{D}$ is block diagonal, that is,

$$\mathbf{U} = \begin{bmatrix} \mathbf{D}_1^{\frac{1}{2}} & & & \\ & \mathbf{D}_2^{\frac{1}{2}} & & \\ & & \ddots & \\ & & & \mathbf{D}_n^{\frac{1}{2}} \end{bmatrix} \times \begin{bmatrix} \mathbf{I}_{2\times2} & \tilde{\mathbf{u}}_{12} & \tilde{\mathbf{u}}_{13} & \cdots & \tilde{\mathbf{u}}_{1n} \\ & \mathbf{I}_{2\times2} & \tilde{\mathbf{u}}_{23} & \cdots & \tilde{\mathbf{u}}_{2n} \\ & & \mathbf{I}_{2\times2} & \cdots & \tilde{\mathbf{u}}_{3n} \\ & & & \ddots & \vdots \\ & & & & \mathbf{I}_{2\times2} \end{bmatrix} \quad (20)$$

where $\mathbf{D}_i^{\frac{1}{2}}$ and $\tilde{\mathbf{u}}_{jk}$ are 2×2 matrices and $\mathbf{I}_{2\times2}$ is a 2×2 identity matrix.

Suppose that a $2 \times n$ matrix $\tilde{\mathbf{p}}$, which correspond to a pair of statistically coupled measurements in $[\mathbf{H} \mid \mathbf{z}]$, should undergo rotations with a $2 \times n$ submatrix $\tilde{\mathbf{u}}$ of $\tilde{\mathbf{U}}$ in order to zero out block $\tilde{\mathbf{p}}_i$.

$$\begin{array}{l} \tilde{\mathbf{u}} = [\begin{array}{cccccccc} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{D}^{\frac{1}{2}} & \cdots & \mathbf{D}^{\frac{1}{2}}\tilde{\mathbf{u}}_k & \cdots & \mathbf{D}^{\frac{1}{2}}\tilde{\mathbf{u}}_{n+1} \end{array}] \\ \tilde{\mathbf{p}} = [\begin{array}{cccccccc} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{W}^{\frac{1}{2}}\tilde{\mathbf{p}}_i & \cdots & \mathbf{W}^{\frac{1}{2}}\tilde{\mathbf{p}}_k & \cdots & \mathbf{W}^{\frac{1}{2}}\tilde{\mathbf{p}}_{n+1} \end{array}] \end{array} \quad (21)$$

After a single block rotation, the row vectors take the form

$$\begin{array}{l} \tilde{\mathbf{u}}' = [\begin{array}{cccccccc} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{D}'^{\frac{1}{2}} & \cdots & \mathbf{D}'^{\frac{1}{2}}\tilde{\mathbf{u}}'_k & \cdots & \mathbf{D}'^{\frac{1}{2}}\tilde{\mathbf{u}}'_{n+1} \end{array}] \\ \tilde{\mathbf{p}}' = [\begin{array}{cccccccc} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{W}'^{\frac{1}{2}}\tilde{\mathbf{p}}'_k & \cdots & \mathbf{W}'^{\frac{1}{2}}\tilde{\mathbf{p}}'_{n+1} \end{array}] \end{array} \quad (22)$$

In analogy with the scalar form, the sequence of block rotations successively annihilates all nonzero blocks of $\tilde{\mathbf{p}}$. In this process, matrices $\tilde{\mathbf{U}}$, $\tilde{\mathbf{c}}$ and $\tilde{\mathbf{e}}$ are also updated.

Although the weighting mechanism employed in the blocked version of Givens rotations is analogous to its scalar form, it exhibits the distinctive advantage of enabling the consideration of the statistical correlation involving each new pair of measurements being processed. This is accomplished by defining the 2×2 measurement weighting factor as:

$$\mathbf{W} = \begin{bmatrix} \sigma^2_{\mathcal{R}e} & c_{Re,Im} \\ c_{Im,Re} & \sigma^2_{Im} \end{bmatrix}^{-1} \tag{23}$$

where $\sigma^2_{\mathcal{R}e}$ and $\sigma^2_{\mathcal{I}m}$ are the variances of the real and imaginary parts of the phasor measurement being processed and $c_{Re,Im} = c_{Im,Re}$ are the corresponding covariance.

Similarly, the correlation between pairs of voltage components in the *a priori* state information vector can be easily taken into account. To achieve this, the generalized weighting factor $\mathbf{D}$ of the block triangular matrix in (20) should be initialized as:

$$\mathbf{D}^{(0)} = \begin{bmatrix} \bar{\sigma}^2_{\mathcal{R}e} & \bar{c}_{\mathcal{R}e,\mathcal{I}m} \\ \bar{c}_{\mathcal{I}m,\mathcal{R}e} & \bar{\sigma}^2_{\mathcal{I}m} \end{bmatrix}^{-1} \tag{24}$$

where entries of matrix $\mathbf{D}^{(0)}$ are similar to those in (23) but refer to *a priori* state information. Vector $\tilde{\mathbf{c}}$ of (18) should contain the a priori state information values ordered as a sequence of bus voltage real and imaginary parts, that is:

$$\tilde{\mathbf{c}}_1 = \bar{\mathbf{x}}_{priori} \tag{25}$$

IV. Enhanced Two-Stage PSSE

The hybrid state estimator employed in this paper builds upon the architecture proposed in [5], which is depicted in Fig. 1. Its first estimation stage is simply a conventional state estimator based on SCADA measurements, with no extra restrictions imposed. The output of this module, which comprises the estimated state vector and the corresponding estimation error covariance matrix (6), is treated as *a priori* information by the second estimation stage. The latter processes phasor measurements only, and consists of an orthogonal estimator based on 3M *Blocked* Givens rotations. As shown in Subsection III-B, that version of Givens rotations is able to process *a priori* information without any extra computational cost. A particular attractive feature of such strategy is that it maintains intact the structure already in place with existing SCADA-based state estimators.

A. Improvements to phasor measurement processing

Although relying in the previously proposed architecture described in Fig. 1, two relevant enhancements to the estimation process are introduced in this paper, both of them related to the second estimation stage. They are:

1) The relationships between phasor measurements and state variables is converted to rectangular coordinates [4], so that the resulting measurement model becomes *linear.* As a consequence, the solution of the estimation problem can be obtained through a direct, non-iterative algorithm, with obvious computational advantages;
2) The orthogonal algorithm is replaced by the *block* form of 3M Givens rotations described in Subsection III-C. As described in the latter, this enables the estimator to take into account the correlation between the real and imaginary parts of both PMU measurements and *a priori* information, thus preserving the statistical properties of the solution.

Since the properties connected to item 2 above have been discussed in Subsection III-C, in the remaining of this section attention is focused on the issues related to item 1.

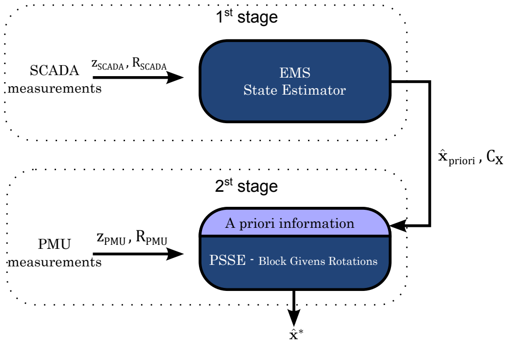


Fig. 1. Two-stage estimation strategy

B. PSSE in rectangular coordinates considering PMU measurements

The second stage processes only PMU measurements in rectangular coordinates and embeds a priori information obtained from the first stage. This choice of coordinates system improves computational efficiency, since real and imaginary parts of voltage and currents are linearly related to the states. To outline this, let y_{pq} be the series admittance of the branch connecting buses p and q and y_{p0} be the shunt admittance at bus p, so that:

$$\begin{aligned} y_{pq} &= g_{pq} + jb_{pq} \\ y_{p0} &= jb_{p0} \end{aligned} \tag{26}$$

Now consider a set of two distinct PMU measurements taken at bus p, namely, phasor current I_{pq} and phasor voltage V_p. The corresponding measurement model takes the form:

$$\begin{bmatrix} V_p^r \\ V_p^i \\ I_{pq}^r \\ I_{pq}^i \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ g_{pq} & (b_{pq} - b_{p0}) & -g_{pq} & b_{pq} \\ (b_{pq} + b_{p0}) & g_{pq} & -b_{pq} & -g_{pq} \end{bmatrix} \begin{bmatrix} x_p^r \\ x_p^i \\ x_q^r \\ x_q^i \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{bmatrix} \tag{27}$$

where superscript r and i stand for real and imaginary parts, respectively.

Since phasor measurements are physically taken in polar coordinates, a change of coordinates is needed in order to employ (27). This involves two steps. The first one is simply the conversion of the measured values $|z_i|\angle\theta_i$ (voltage or current) to rectangular form:

$$\begin{cases} z_i^r = |z_i| cos(\theta_i) \\ z_i^i = |z_i| sin(\theta_i) \end{cases} \tag{28}$$

The corresponding error covariance matrix has also to be converted to the new coordinates. Assuming that z_i and θ_i measurement errors are independent, one clearly sees from (28) that such a step involves two nonlinear functions of two independent random variables [13]. Equations (28) can be linearized, yielding the following relationships between the incremental measurement components in polar and rectangular coordinates:

$$\begin{bmatrix} \Delta z_i^r \\ \Delta z_i^i \end{bmatrix} = \begin{bmatrix} \dfrac{\partial z_i^r}{\partial |z_i|} & \dfrac{\partial z_i^r}{\partial \theta_i} \\ \dfrac{\partial z_i^i}{\partial |z_i|} & \dfrac{\partial z_i^i}{\partial \theta_i} \end{bmatrix} \begin{bmatrix} \Delta |z_i| \\ \Delta \theta_i \end{bmatrix} \tag{29}$$

Defining the transformation matrix $\mathbf{M}$ as

$$\mathbf{M}(|z_i|,\theta_i) = \begin{bmatrix} \frac{\partial z_i^r}{\partial |z_i|} & \frac{\partial z_i^r}{\partial \theta_i} \\ \frac{\partial z_i^i}{\partial |z_i|} & \frac{\partial z_i^i}{\partial \theta_i} \end{bmatrix} = \begin{bmatrix} cos(\theta_i) & -|z_i|sin(\theta_i) \\ sin(\theta_i) & |z_i|cos(\theta_i) \end{bmatrix}, \quad (30)$$

the relationship between the covariance matrices in rectangular and polar forms is given by [13]:

$$\mathbf{R}_{rect} = \mathbf{M}\mathbf{R}_{pol}\mathbf{M}^t \quad (31)$$

Equation (31) shows that, even assuming statistical independence between measurement polar components, $\mathbf{R}_{rect}$ will be a full 2×2 block. When a set of phasor measurements are considered, this leads to the block diagonal structure represented in (17). Another important matrix structure characteristic can observed from $\mathbf{H}$ in (27). When states and measurements are ordered as in (27), a 2×2 block structure results, so that the measurement model can be rewritten as:

$$\begin{bmatrix} \mathbf{V_p} \\ \mathbf{I_{pq}} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{2\times2} & \\ \mathbf{B}_1 & \mathbf{B}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_p \\ \mathbf{x}_q \end{bmatrix} + \begin{bmatrix} \eta_{1,2} \\ \eta_{3,4} \end{bmatrix} \quad (32)$$

The last issue that remains to be discussed is the processing of the results from the first stage as *a priori* information by the second stage. Those results are usually in polar coordinates, so that the same transformations given by (28) and (31) apply. Accordingly, the SCADA-based estimates are converted into rectangular form and compose the *a priori* state information vector in (25) to be processed next by the second estimation module. In addition, the error covariance matrix of the first stage in rectangular coordinates is given by:

$$\mathbf{C}_{\mathbf{x},rect} = \mathbf{M}\mathbf{C}_{\mathbf{x},polar}\mathbf{M}^t \quad (33)$$

Notice that $\mathbf{C}_{\mathbf{x},polar}$ is generally a full matrix. In order to accommodate $\mathbf{C}_{\mathbf{x},rect}$ into the block Givens rotations framework, the assumption is made that the cross correlation between state variables associated to two distinct buses can be neglected when compared with the 2×2 covariance blocks of each bus complex voltage. This results in a block submatrix $\tilde{\mathbf{C}}_{\mathbf{x},rect}$ whose structure is similar to that of matrix $\tilde{\mathbf{R}}$ in (17). Considering the developments in Subsection III-C, it is easy to conclude that the 2×2 blocks of $\tilde{\mathbf{C}}_{\mathbf{x},rect}$ define the initial factor $\mathbf{D}^{(0)}$ of (24), which weighs the block triangular matrix of the generalized rotations.

Finally, it is important to notice that it is the choice of the *block* form of Givens rotations that makes it possible to consider the off diagonal entries of $\mathbf{C}_{\mathbf{x},rect}$, thus preserving important statistical properties of the SCADA-based estimates provided by the first estimation module.

V. Simulation results

In order to evaluate the gains in accuracy provided by the proposed strategy based on blocked 3M Givens rotations, several simulations have been carried out on the *IEEE* 14-bus and 30-bus test systems. The starting point for all simulations is a power flow study which provides the "true" value for the state and network variables. Measurements are generated by adding random Gaussian distributed errors with zero mean and standard deviations which are a function of the specified meter accuracies. Measurement errors are not allowed to exceed $\pm3\sigma$ in order to avoid adding bad data to the measurement set. It is assumed that the accuracy level for SCADA measurements is 1% and for PMU measurements is 0.1%, for both magnitude and angle. An orthogonal scalar state estimator is employed to process the SCADA measurements, although there is no restriction concerning the algorithm used at this stage. The metering schemes are such that the test systems are fully observable with respect to SCADA measurements. The second stage processes phasor measurements from a number of PMU units placed in the system. It is assumed that each PMU measures the complex voltage at the bus where it is installed and the complex currents on all branches incident to it. The PMU measurement sets themselves do not necessarily ensure network observability. It is assumed that the voltage phasor is monitored at the reference bus, which is bus 1 for both networks.

For both test systems, fifty simulations are performed, each of them based on different measurement values obtained by using random seeds to generate the measurement errors. Each measurement set is then submitted to both the scalar and blocked versions of the 3M Givens *APSI* estimator.

The metering scheme used for the simulations with the *IEEE* 14-bus system is detailed in the one-line diagram in Fig. 2. A similar diagram is not shown for the 30-bus test system due to space limitations. Table I specifies the number of measurements of each type which compose the metering scheme, for each test system.

Fig. 3 present the mean absolute errors for both voltage magnitude and angle at each bus for the 14-bus test system. Each point plotted in both figures is obtained by applying the formula:

$$\bar{x} = \frac{\sum_{k=1}^{n_S} \left|\hat{x}_k^{method} - x_k^{true}\right|}{n_S} \quad (34)$$

where x stands for either the voltage magnitude V_i or the voltage angle θ_i at bus i; n_S is the number of performed simulations (equal to 50 in this case); superscript *"true"* refers to values obtained from the power flow study, and superscript *"method"* refers to values provided by either the scalar or the blocked version of the 3M Givens *APSI* estimator. Results for the 30-bus network are also obtained (34) and are shown in Fig. 4.

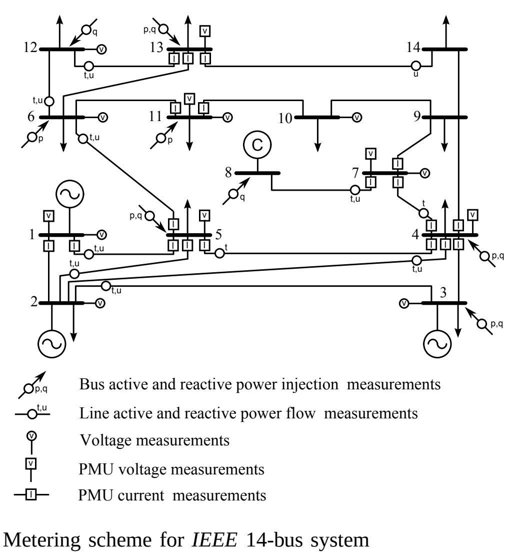


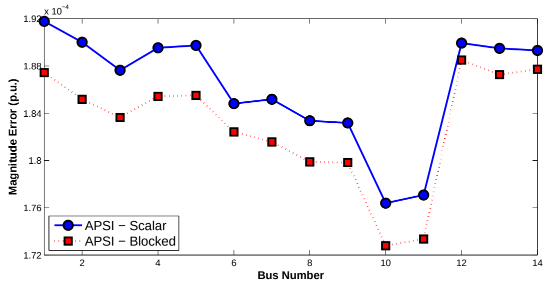
Fig. 2. Metering scheme for *IEEE* 14-bus system

Results in Fig. 3 and 4 clearly show that the improved modeling of the statistical properties provided by the blocked

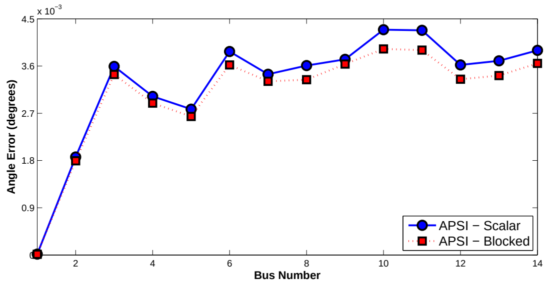
TABLE I
METERING SCHEME COMPOSITION FOR BOTH TEST SYSTEMS

	SCADA					PMU	
Meas. Type	P	Q	$\|V\|$	t	u	$\dot{V}$	$\dot{I}$
14-BUS	6	6	8	10	9	6	19
30-BUS	15	15	14	27	27	10	19

version of the 3M Givens rotations-based APSI estimator reflects itself on the accuracy of the estimates. In all plots, the mean absolute errors computed for the proposed method are consistently smaller than those corresponding to an algorithm also based on the 3M Givens rotations, but employing a scalar measurement weighting scheme.



(a)



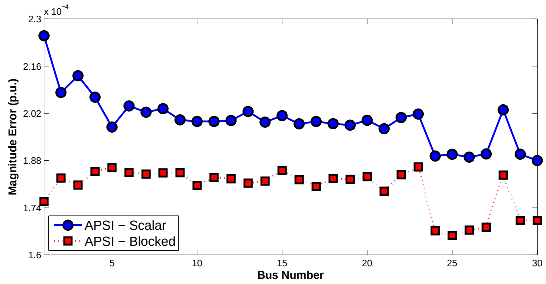
(b)

Fig. 3. Mean absolute errors of (a) voltage magnitudes and (b) voltage angles for the IEEE 14-bus system

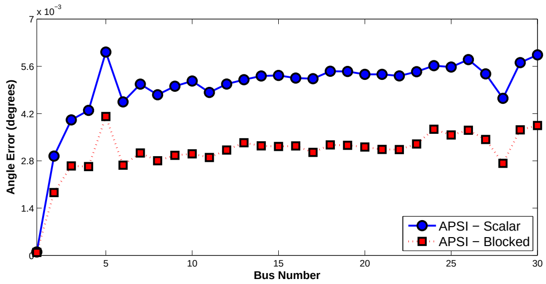
VI. CONCLUSIONS

This paper proposes an alternative method to incorporate phasor measurements to power system state estimation. It is based on a two-stage process which imposes no restriction to existing SCADA-based estimators. The results provided by the latter are embedded into the second stage as *a priori* information and then combined with the available phasor measurements. In the second stage, all data is processed in rectangular coordinates, in order to benefit from the resulting linear relationships between measurements and states. To ensure that such a change of coordinates will not adversely affect the statistical properties of the solution, a novel measurement weighting blocked scheme based on Givens orthogonal rotations is proposed in this paper. Extensive simulation results carried out with two test systems confirm that the proposed method provides more accurate estimates than conventional scalar measurement weighting schemes.

Additional topics related to the proposed estimator deserve further research efforts, such as a more careful appraisal of its computational requirements and the impact of the enhanced



(a)



(b)

Fig. 4. Mean absolute errors of (a) voltage magnitudes and (b) voltage angles for the IEEE 30-bus system

statistical modeling provided by the novel estimation architecture on bad data processing.

REFERENCES

[1] Y. Yoon, "Study of the utilization and benefits of phasor measurement units for large scale power system state estimation," Ph.D. dissertation, Texas A&M University, 2005.

[2] T. Baldwin, L. Mili, J. Boisen, M.B., and R. Adapa, "Power system observability with minimal phasor measurement placement," *Power Systems, IEEE Transactions on*, vol. 8, no. 2, pp. 707 –715, may 1993.

[3] J. Chen and A. Abur, "Placement of pmus to enable bad data detection in state estimation," *Power Systems, IEEE Transactions on*, vol. 21, no. 4, pp. 1608 –1615, nov. 2006.

[4] M. Zhou, V. Centeno, J. Thorp, and A. Phadke, "An alternative for including phasor measurements in state estimators," *Power Systems, IEEE Transactions on*, vol. 21, no. 4, pp. 1930–1937, 2006.

[5] A. Simões Costa and A. Albuquerque, "A two-stage orthogonal estimator to incorporate phasor measurements into power system real-time modeling," *Power Systems Computation Conference (PSCC)*, 2011.

[6] R. Nuqui and A. Phadke, "Hybrid linear state estimation utilizing synchronized phasor measurements," in *Power Tech, 2007 IEEE Lausanne*, july 2007, pp. 1665 –1669.

[7] F. Schweppe and J. Wildes, "Power system static-state estimation, part i: Exact model," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-89, no. 1, pp. 120 –125, jan. 1970.

[8] A. Monticelli, *State estimation in electric power systems: a generalized approach*. Springer Netherlands, 1999, vol. 507.

[9] A. Abur and A. Exposito, *Power system state estimation: theory and implementation*. CRC, 2004, vol. 24.

[10] A. Simões Costa and V. Quintana, "An orthogonal row processing algorithm for power system sequential state estimation," *Power Apparatus and Systems, IEEE Transactions on*, no. 8, pp. 3791–3800, 1981.

[11] W. Gentleman, "Least squares computations by givens transformations without square roots," *IMA Journal of Applied Mathematics*, vol. 12, no. 3, p. 329, 1973.

[12] S. Hammarling, "A note on modifications to the givens plane rotation," *IMA Journal of Applied Mathematics*, vol. 13, no. 2, pp. 215–218, 1974.

[13] A. Papoulis and S. Pillai, *Probability, random variables and stochastic processes with errata sheet*. McGraw Hill Higher Education, 2002.