Fontes Renováveis Não-Convencionais

Parte II

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Power in the Wind

- Consider the kinetic energy of a "packet" of air with mass m moving at velocity v

$$\text{KE} = \frac{1}{2} m v^2$$

- Divide by time and get power

$$\text{Power through area A} = \frac{1}{2}\left(\frac{m \text{ passing though A}}{t}\right)v^2$$

- The mass flow rate is (ρ is air density)

$$\dot{m} = \frac{m \text{ passing though A}}{t} = \rho \text{A} v$$

Power in the Wind

Combining previous equations,

$$\text{Power through area A} = \frac{1}{2}(\rho A v)v^2$$

$$\boxed{P_W = \frac{1}{2}\rho A v^3} \quad (6.4) \quad \Leftarrow \quad \text{Power in the wind}$$

P_W (Watts) = power in the wind

ρ (kg/m^3)= air density (1.225kg/m^3 at 15˚C and 1 atm)

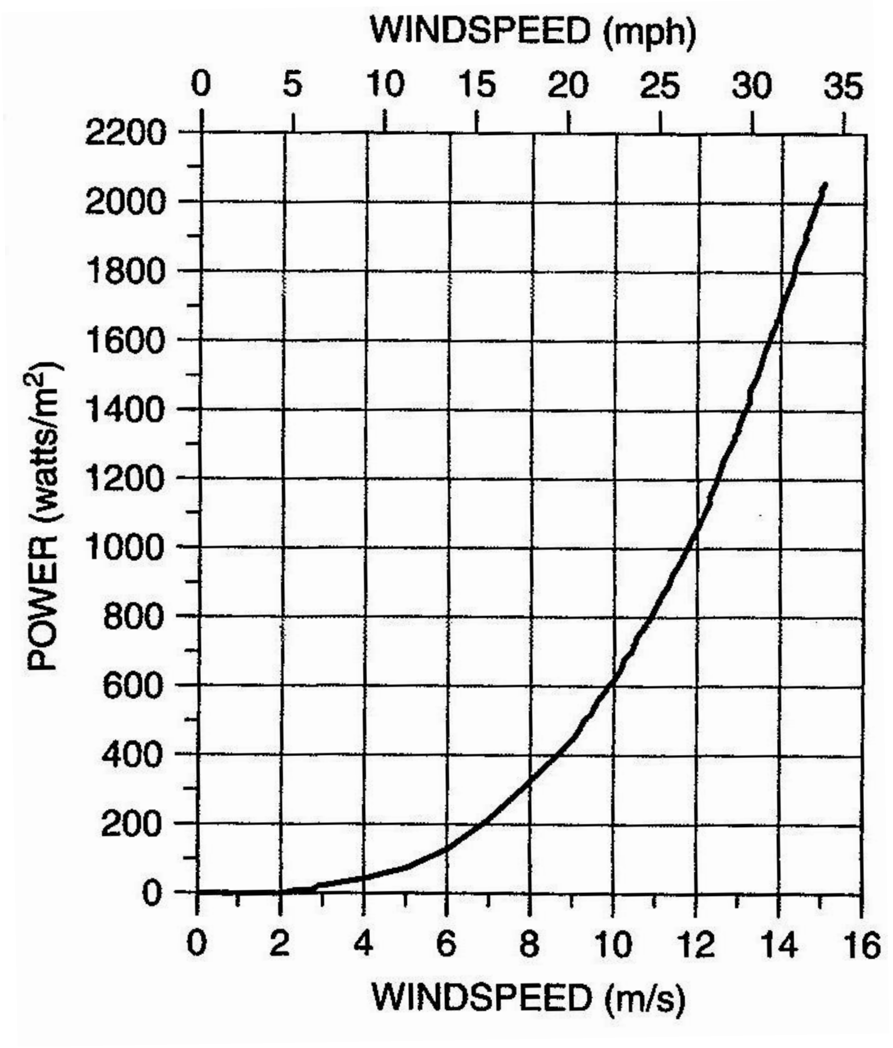
A (m^2)= the cross-sectional area that wind passes through

v (m/s)= wind speed normal to A (1 m/s = 2.237 mph)

(3,6 km/h)

Power in the Wind (for reference solar is about 600 w/m^2 in summer)

- Power increases with the cube of wind speed
- Doubling the wind speed increases the power by eight
- Energy in 1 hour of 20 mph winds is the same as energy in 8 hours of 10 mph winds
- Nonlinear, so we cannot use average wind speed



Power in the Wind

$$P_W = \frac{1}{2}\rho A v^3$$

- Power in the wind is also proportional to A
- For a conventional HAWT, $A = (\pi/4)D^2$, so wind power is proportional to the blade diameter squared
- Cost is somewhat proportional to blade diameter
- This explains why larger wind turbines are more cost effective (plus, as we shall see, because they are higher, the winds are stronger)

Example: Energy in 1 m^2 of Wind

$$\text{Energy} = \frac{1}{2}\rho A v^3 \Delta t$$

- 100 hours of 6 m/s winds

$$\text{Energy} = \frac{1}{2}\left(1.225\ \text{kg/m}^3\right)\left(1\text{m}^2\right)\left(6\ \text{m/s}\right)^3 100\ \text{h} = \boxed{13{,}230\ \text{Wh}}$$

- 50 hours of 3 m/s winds and 50 hours of 9 m/s winds - *the average wind speed is 6 m/s

$$\text{Energy (3 m/s)} = \frac{1}{2}\left(1.225\ \text{kg/m}^3\right)\left(1\text{m}^2\right)\left(3\ \text{m/s}\right)^3 50\ \text{h} = 827\ \text{Wh}$$

$$\text{Energy (9 m/s)} = \frac{1}{2}\left(1.225\ \text{kg/m}^3\right)\left(1\text{m}^2\right)\left(9\ \text{m/s}\right)^3 50\ \text{h} = 22{,}326\ \text{Wh}$$

Don’t use average wind speed!

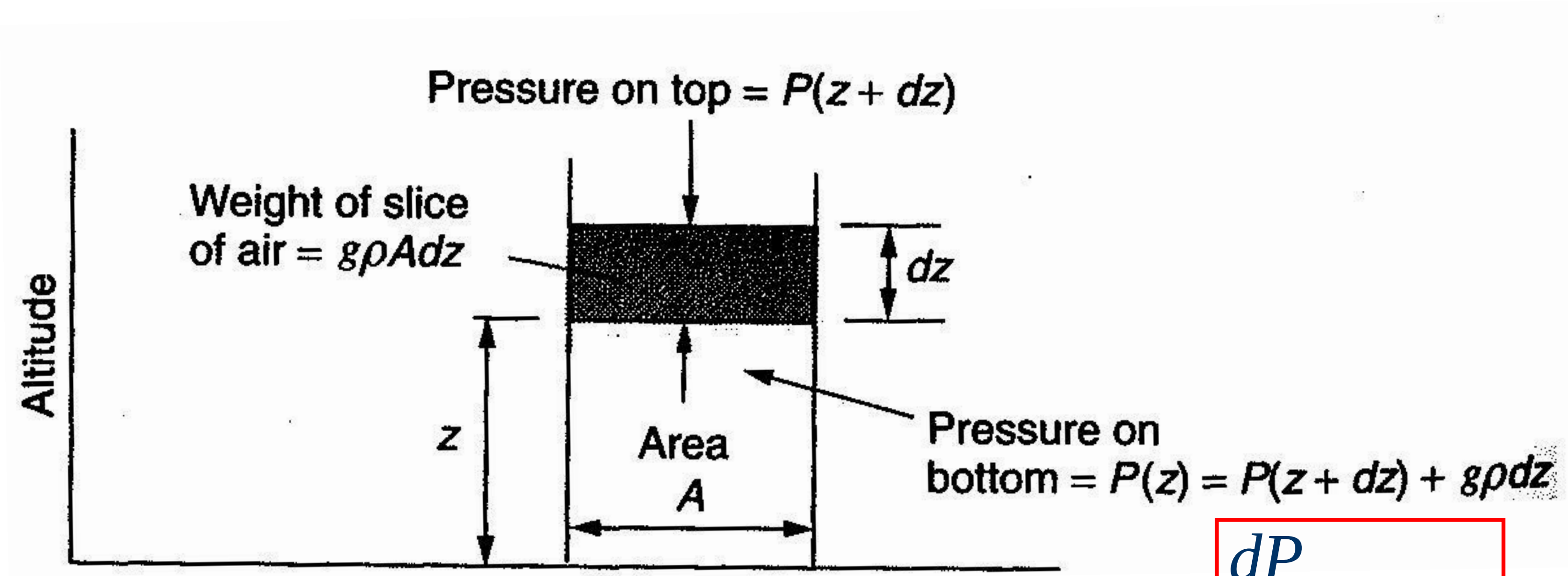
$$\text{total} = \boxed{23{,}152\ \text{Wh}}$$

Air Density for Different Temperatures and Pressures

$$\boxed{\rho = \frac{P \cdot \mathrm{M.W.} \cdot 10^{-3}}{RT}}$$

- P = absolute pressure (atm)
- M.W. = molecular weight of air (g/mol) = 28.97 g/mol
- T = absolute temperature (K)
- R = ideal gas constant = $8.2056 \cdot 10^{-5} \cdot m^3 \cdot atm \cdot K^{-1} \cdot mol^{-1}$
- Air density is greater at lower temperatures
  - For example, in comparing 90º F (305 K) (32°C) to 10º F 265.3 K) (−12°C), ratio is about 1.15

Air Density Altitude Correction



$$dP = P(z+dz) - P(z) = -g\,\rho dz \quad \text{==>} \quad \boxed{\frac{dP}{dz} = -\rho g}$$

We get a differential equation in terms of pressure:*

$$\frac{dP}{dz} = -1.185 \cdot 10^4 P \quad \Rightarrow \quad \boxed{P = 1\ \text{atm} \cdot e^{-1.185 \cdot 10^4 H}}$$

where H is in meters

* Assuming T constant with altitude

Air Density Temperature and Altitude Impacts

- Variation in density with respect to temperature and altitude is given by

$$\boxed{\rho = \frac{353.1\exp(-0.0342\,z\,/\,T)}{T} \quad \text{kg/m}^3}$$

where T is in kelvins (K) and z is in meters above sea level

- With z=0, T=273.16+15 then $\rho = 1.225 \text{kg/m}^3$
  - With z=200, T= 273.16+35, then ρ = ~~1.225kg/m³~~ 1.120 kg/m³ (about 91% of sea level, 15 degree C value)

Impact of Elevation and Earth's Roughness on Windspeed

- Since power increases with the cube of wind speed, we can expect a significant economic impact from even a moderate increase in wind speed
- There is a lot of friction in the first few hundred meters above ground – smooth surfaces (like water) are better
- Wind speeds are greater at higher elevations – tall towers are better
- Forests and buildings slow the wind down a lot

Characterization of Elevation and Earth's Roughness on Wind Speed

$$\frac{v}{v_0} = \left(\frac{H}{H_0} \right)^{\alpha}$$

- α = friction coefficient – given in Table 6.3
- v = wind speed at height H
- v_0 = wind speed at height H_0 (H_0 is usually 10 m)
- Typical value of α in open terrain is 1/7
- For a large city, $\alpha = 0.4$; for small town, $\alpha = 0.3$, for high crops, , $\alpha = 0.2$, for calm water or hard ground, $\alpha = 0.1$

Friction coefficient

TABLE 6.3 Friction Coefficient for Various Terrain Characteristics

Terrain Characteristics	Friction Coefficient α
Smooth hard ground, calm water	0.10
Tall grass on level ground	0.15
High crops, hedges and shrubs	0.20
Wooded countryside, many trees	0.25
Small town with trees and shrubs	0.30
Large city with tall buildings	0.40

Impact of Elevation and Earth's Roughness on Wind speed

- Alternative formulation (used in Europe)

$$\frac{v}{v_0} = \frac{\ln(H/l)}{\ln(H_0/l)}$$

- l is the "roughness length" – given in Table 7.2
- Note that both equations are just approximations of the variation in wind speed due to elevation and roughness– the best thing is to have actual measurements

Roughness length

TABLE 6.4 Roughness Classifications for Use in (6.16)

Roughness Class	Description	Roughness Length $z(m)$
0	Water surface	0.0002
1	Open areas with a few windbreaks	0.03
2	Farm land with some windbreaks more than 1 km apart	0.1
3	Urban districts and farm land with many windbreaks	0.4
4	Dense urban or forest	1.6

Impact of Elevation and Earth's Roughness on Power in the Wind

- Combining earlier equations we get

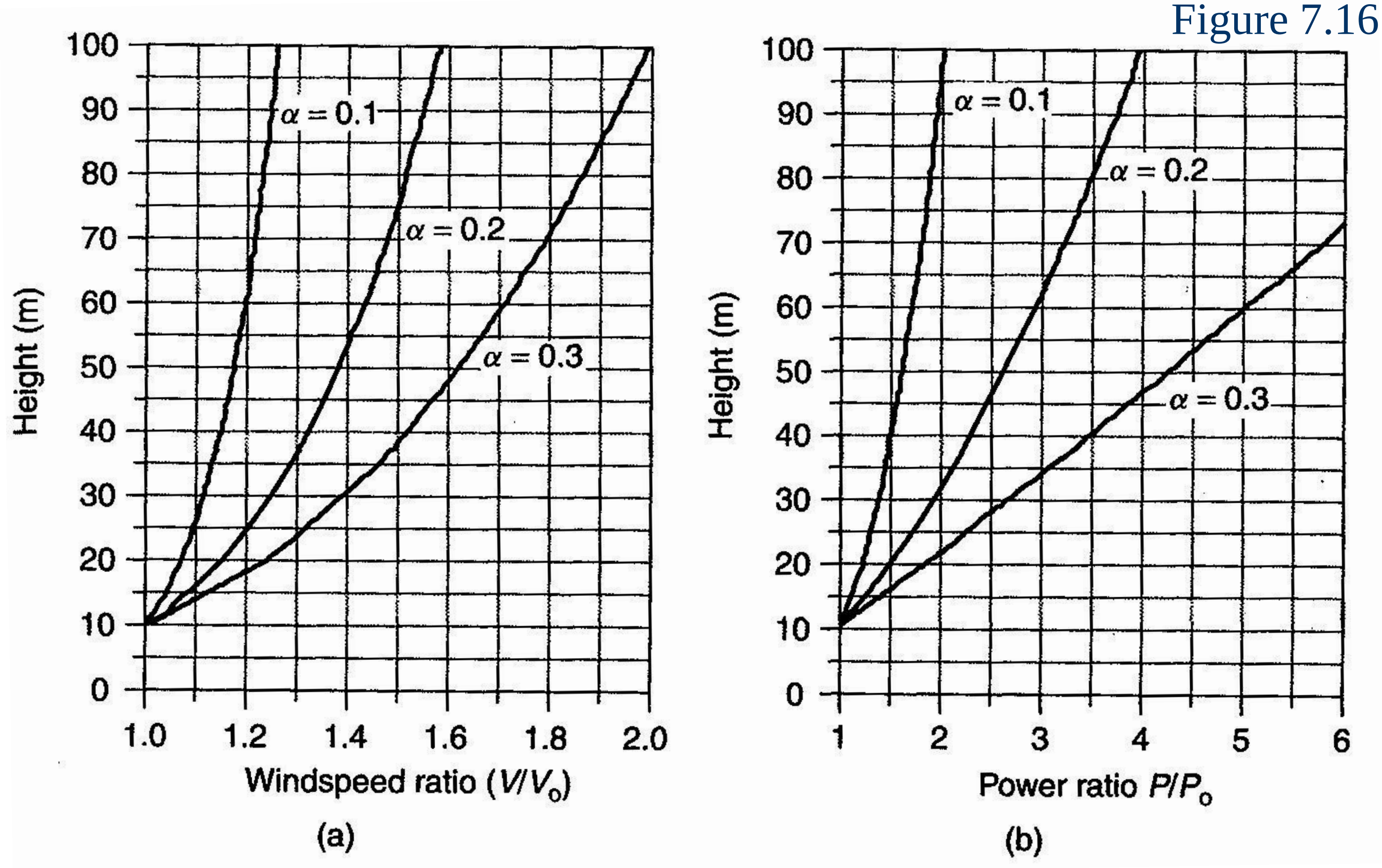
$$\frac{P_W}{P_{W_0}} = \left(\frac{v}{v_0}\right)^3 = \left(\frac{H}{H_0}\right)^{3\alpha}$$

- The other constants in the power in the wind equation are the same, so they just cancel:

$$\frac{P_W}{P_{W_0}} = \frac{\cancel{\frac{1}{2}}\cancel{\rho}\cancel{A}v^3}{\cancel{\frac{1}{2}}\cancel{\rho}\cancel{A}v_0^{\,3}}$$

Impact of Elevation and Earth's Roughness on Windspeed

Figure 7.16



For a small town, windspeed at 100 m is twice that at 10 m

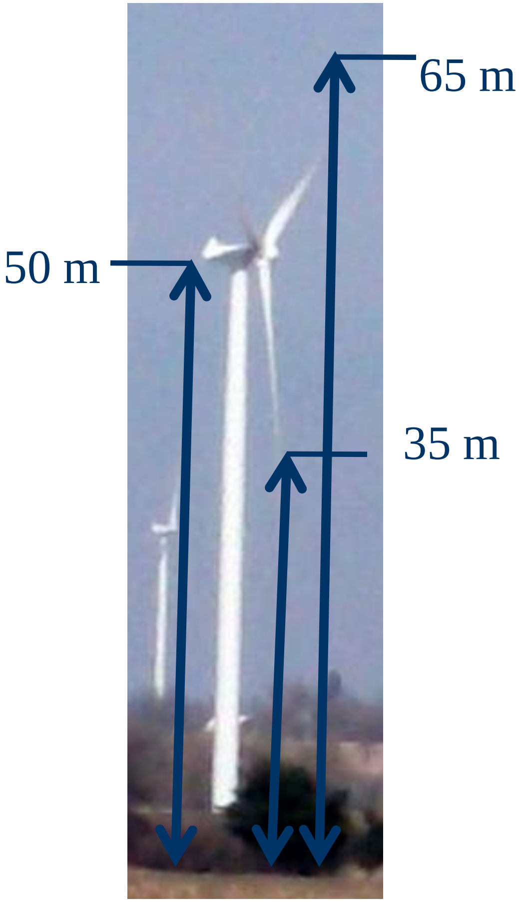
Areas with smoother surfaces have less variation with height

Example Rotor Stress

- Wind turbine with hub at 50-m and a 30-m diameter rotor, $\alpha = 0.2$
- Find the ratio of power in the wind at highest point to lowest point

$$\frac{P}{P_0} = \left(\frac{65}{35}\right)^{3\cdot 0.2} = 1.45$$

- Power in the wind at the top of the blades is 45% higher!



Picture may not be to scale

Maximum Rotor Efficiency

- Two extreme cases, and neither makes sense-
  - Downwind velocity is zero – turbine extracted all of the power
  - Downwind velocity is the same as the upwind velocity – turbine extracted no power

- Albert Betz 1919 - There must be some ideal slowing of the wind so that the turbine extracts the maximum power

Maximum Rotor Efficiency

- Constraint on the ability of a wind turbine to convert kinetic energy in the wind into mechanical power
- Think about wind passing though a turbine- it slows down and the pressure is reduced so it expands

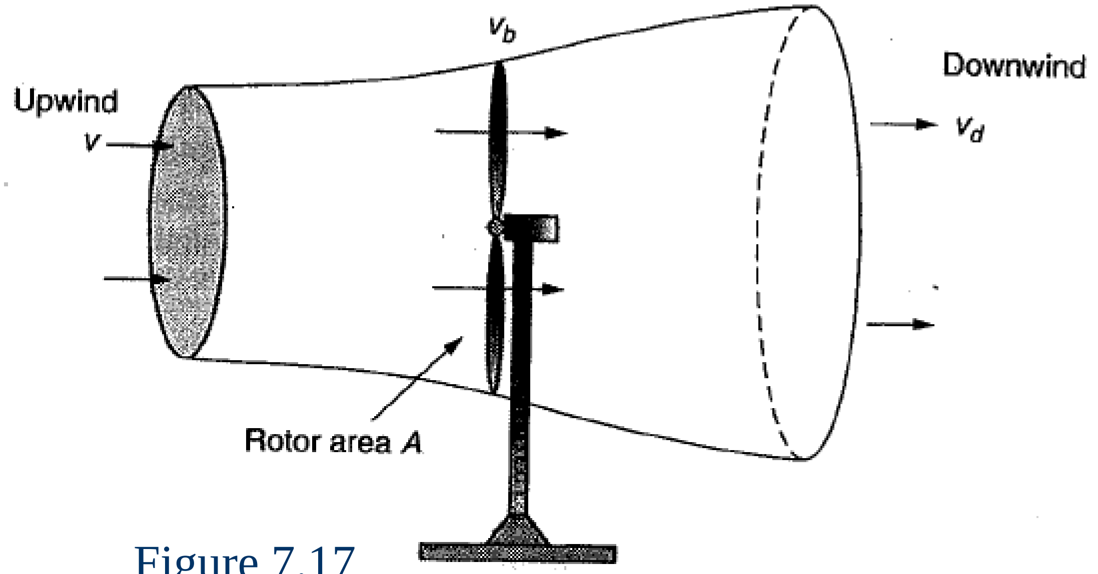


Figure 7.17

Power Extracted by The Blades

$$P_b = \frac{1}{2}\dot{m}\left(v^2 - {v_d}^2\right) \qquad (7.21)$$

- $\dot{m}$ = mass flow rate of air within stream tube
- v = upwind undisturbed wind speed
- v_d = downwind wind speed

- From the difference in kinetic energy between upwind and downwind air flows

Determining Mass Flow Rate

- Easiest to determine at the plane of the rotor because we know the cross sectional area A
- Then, the mass flow rate is

$$\dot{m} = \rho A v_b \qquad (7.22)$$

- Assume the velocity through the rotor v_b is the average of upwind velocity v and downwind velocity v_d

$$v_b = \frac{v + v_d}{2} \Rightarrow \dot{m} = \rho A \left(\frac{v + v_d}{2} \right)$$

Power Extracted by the Blades

- Then

$$P_b = \frac{1}{2}\rho A\left(\frac{v+v_d}{2}\right)\left(v^2 - {v_d}^2\right) \qquad (7.23)$$

- Define

$$\boxed{\lambda = \frac{v_d}{v}}, \text{ will be less than 1.0} \qquad (7.24)$$

- Then substituting for v_d to get the power extracted

$$P_b = \frac{1}{2}\rho A\left(\frac{v+\lambda v}{2}\right)\left(v^2 - \lambda^2 v^2\right) \qquad (7.25)$$

Power Extracted by the Blades

$$P_b = \frac{1}{2}\rho A \underbrace{\left(\frac{v+\lambda v}{2}\right)\left(v^2 - \lambda^2 v^2\right)}$$

$$\left(\frac{v+\lambda v}{2}\right)\left(v^2 - \lambda^2 v^2\right) = \frac{v^3}{2} - \frac{\lambda^2 v^3}{2} + \frac{\lambda v^3}{2} - \frac{\lambda^3 v^3}{2}$$

$$= \frac{v^3}{2}\left[\left(1+\lambda\right) - \lambda^2\left(1+\lambda\right)\right]$$

$$= \frac{v^3}{2}\left[\left(1+\lambda\right)\left(1-\lambda^2\right)\right]$$

$$\Rightarrow \quad P_b = \underbrace{\frac{1}{2}\rho A v^3}_{P_W = \text{Power in the wind}} \cdot \underbrace{\frac{1}{2}\left[\left(1+\lambda\right)\left(1-\lambda^2\right)\right]}_{C_P = \text{Rotor efficiency}}$$

Maximum Rotor Efficiency

- Find the wind speed ratio λ that maximizes the rotor efficiency, C_P
- From the previous slide

$$C_P = \frac{1}{2}\left[(1+\lambda)(1-\lambda^2)\right] = \frac{1}{2} - \frac{\lambda^2}{2} + \frac{\lambda}{2} - \frac{\lambda^3}{2}$$

Set the derivative of rotor efficiency to zero and solve for λ:

$$\frac{\partial C_P}{\partial \lambda} = -2\lambda + 1 - 3\lambda^2 = 0$$

$$\frac{\partial C_P}{\partial \lambda} = 3\lambda^2 + 2\lambda - 1 = 0$$

$$\frac{\partial C_P}{\partial \lambda} = (3\lambda - 1)(\lambda + 1) = 0$$

$$\boxed{\lambda = \frac{1}{3}}$$

maximizes rotor efficiency

Maximum Rotor Efficiency

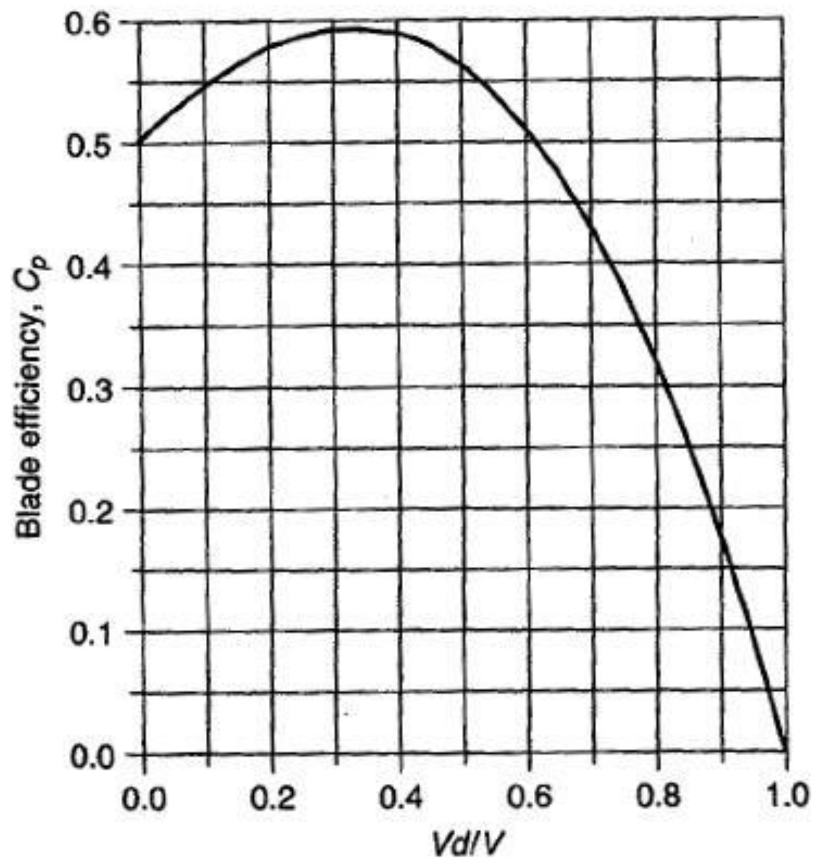
- Plug the optimal value for λ back into C_P to find the maximum rotor efficiency:

$$C_P = \frac{1}{2}\left[\left(1+\frac{1}{3}\right)\left(1-\frac{1}{3^2}\right)\right] = \frac{16}{27} = 59.3\% \qquad (7.29)$$

- The maximum efficiency of 59.3% occurs when air is slowed to 1/3 of its upstream rate
- Called the "Betz efficiency" or "Betz' law"

Maximum Rotor Efficiency

Rotor efficiency C_P vs. wind speed ratio λ



Tip-Speed Ratio (TSR)

- Efficiency is a function of how fast the rotor turns
- Tip-Speed Ratio (TSR) is the speed of the outer tip of the blade divided by wind speed

$$\text{Tip-Speed-Ratio (TSR)} = \frac{\text{Rotor tip speed}}{\text{Wind speed}} = \frac{\text{rpm} \times \pi \text{D}}{60v} \qquad (7.30)$$

- D = rotor diameter (m)
- v = upwind undisturbed wind speed (m/s)
- rpm = rotor speed, (revolutions/min)
- One meter per second = 2.24 miles per hour

Tip-Speed Ratio (TSR)

- TSR for various rotor types
- If blade turns too slow then wind passes through without hitting blade; too fast results in turbulence
- Rotors with fewer blades reach their maximum efficiency at higher tip-speed ratios

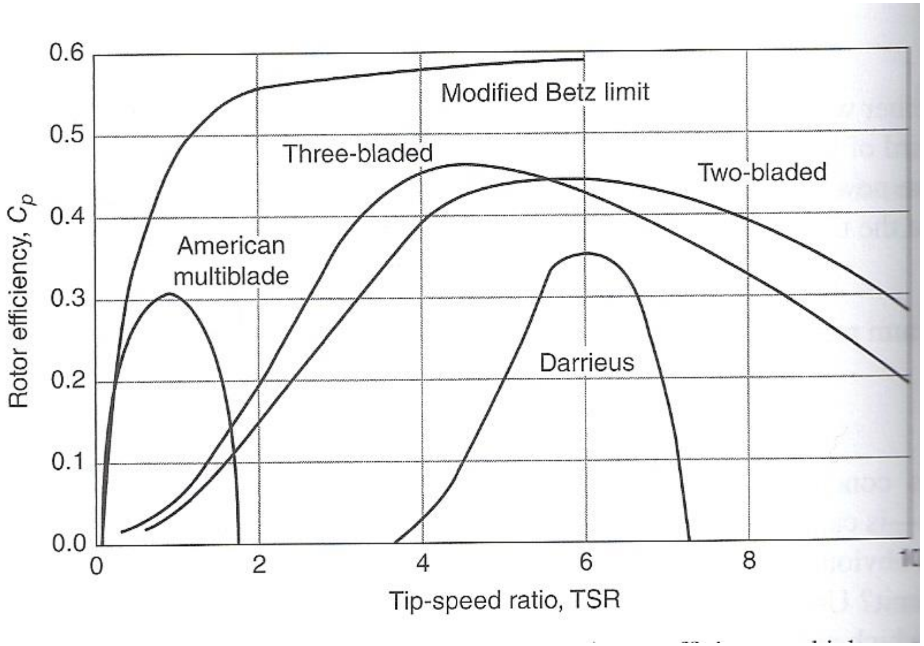


Figure 7.18

A higher TSR is needed when there are fewer blades

Example

- 40-m wind turbine, three-blades, 600 kW, wind speed is 14 m/s, air density is 1.225 kg/m^3

a. Find the rpm of the rotor if it operates at a TSR of 4.0

b. Find the tip speed of the rotor

c. What gear ratio is needed to match the rotor speed to the generator speed if the generator must turn at 1800 rpm?

d. What is the efficiency of the wind turbine under these conditions?

Example

a. Find the rpm of the rotor if it operates at a TSR of 4.0

Rewriting (7.30),

$$\text{rpm} = \frac{\text{Tip-Speed-Ratio (TSR)} \cdot 60v}{\pi \text{D}}$$

$$\text{rpm} = \frac{4.0 \cdot 60\text{sec/min} \cdot 14\text{m/s}}{\pi \cdot 40\text{m/rev}} = 26.7 \text{ rev/min}$$

We can also express this as seconds per revolution:

$$\text{rpm} = \frac{26.7 \text{ rev/min}}{60 \text{ sec/min}} = 0.445 \text{ rev/sec or } 2.24 \text{ sec/rev}$$

Example

b. Tip speed

From (7.30): $$\text{Rotor tip speed} = \frac{\text{rpm} \times \pi D}{60\ \text{sec/min}}$$

$$\text{Rotor tip speed} = (\text{rev/sec}) \times \pi D$$

$$\text{Rotor tip speed} = 0.445\ \text{rev/sec} \cdot \pi 40\ \text{m/rev} \quad = 55.92\ \text{m/s}$$

c. Gear Ratio

$$\text{Gear Ratio} = \frac{\text{Generator rpm}}{\text{Rotor rpm}} = \frac{1800}{26.7} = 67.4$$

Example

d. Efficiency of the complete wind turbine (blades, gear box, generator) under these conditions

From (7.7):

$$P_W = \frac{1}{2}\rho A v^3 = \frac{1}{2}(1.225)\left(\frac{\pi}{4}\cdot 40^2\right)14^3 = 2112 \text{ kW}$$

Overall efficiency:

$$\eta = \frac{600 \text{ kW}}{2112 \text{ kW}} = 28.4\%$$