3.6.1.3 Aplicação do Método da Função Barreira Logarítmica ao FPO - Modelo Linearizado para a Rede

Retomando o exemplo de 4 barras da Seção 3.3 e introduzindo variáveis de folga:

$$\begin{array}{ll} \text{Min} & c(x, p_g) = c_0 + c^T p_g + \frac{1}{2} p_g^T \, Q \, p_g \\ \text{sujeito a:} & -B\,\theta + A_g\, p_g = p_L \\ & p_g + \bar{s}_g = \bar{p}_g \\ & -p_g + \underline{s}_g = -\underline{p}_g \\ & \Gamma\, A\, \theta + \bar{s}_t = \bar{t} \\ & -\Gamma\, A\, \theta + \underline{s}_t = -\underline{t} \end{array}$$

A função Lagrangeana modificada pela adição das funções-barreira logarítmica torna-se:

$$\begin{array}{rl} \mathcal{L}(x, u, \lambda, \pi) = & c_0 + c^T p_g + \frac{1}{2} p_g^T \, Q \, p_g \\ & + \lambda^T (p_L + B\,\theta - A_g\, p_g) \\ & + \bar{\pi}_g^T \, (p_g + \bar{s}_g - \bar{p}_g) \\ & + \underline{\pi}_g^T (-p_g + \underline{s}_g + \underline{p}_g) \\ & + \bar{\pi}_t^T (\Gamma\, A\, \theta + \bar{s}_t - \bar{t}) \\ & + \underline{\pi}_t^T \, (-\Gamma\, A\, \theta + \underline{s}_t + \underline{t}) \\ & - \mu \sum\limits_{i=1}^{n_g} (\ln\, \bar{s}_{gi} + \ln\, \underline{s}_{gi}) \\ & - \mu \sum\limits_{i=1}^{n_l} (\ln\, \bar{s}_{ti} + \ln\, \underline{s}_{ti}) \end{array}$$

ou:

$$\begin{aligned} \mathcal{L}(x, u, \lambda, \pi) \;\; = \;\; & c_0 + c^T p_g + \tfrac{1}{2} p_g^T \, Q \, p_g + \lambda^T (p_L + B\,\theta - A_g\, p_g) \\ & + \begin{bmatrix} \bar{\pi}_g^T & \underline{\pi}_g^T & \bar{\pi}_t^T & \bar{\pi}_t^T \end{bmatrix} \left\{ \begin{bmatrix} I & & I & & & \\ -I & & & I & & \\ & \Gamma A & & & I & \\ & -\Gamma A & & & & I \end{bmatrix} \begin{bmatrix} p_g \\ \theta \\ \bar{s}_g \\ \underline{s}_g \\ \bar{s}_t \\ \underline{s}_t \end{bmatrix} - \begin{bmatrix} \bar{p}_g \\ -\underline{p}_g \\ \bar{t} \\ -\underline{t} \end{bmatrix} \right\} \\ & - \mu \sum_{i=1}^{n_g} (\ln\, \bar{s}_{gi} + \ln\, \underline{s}_{gi}) - \mu \sum_{i=1}^{n_l} (\ln\, \bar{s}_{ti} + \ln\, \underline{s}_{ti}) \end{aligned}$$

Definindo:

$$
\begin{aligned}
s &= [\,\bar{s}_g^T\ \underline{s}_g^T\ \bar{s}_t^T\ \underline{s}_t^T]^T \\
\pi &= [\bar{\pi}_g^T\ \underline{\pi}_g^T\ \bar{\pi}_t^T\ \underline{\pi}_t^T\,]^T \\
F_u &= \begin{bmatrix} I_{n_g} \\ -I_{n_g} \\ 0_{2\,n_l \times\, n_g} \end{bmatrix} \\
F_\theta &= \begin{bmatrix} 0_{2\,n_g \times\, n_l} \\ \Gamma A \\ -\Gamma A \end{bmatrix} \\
L &= \begin{bmatrix} \bar{p}_g & -\underline{p}_g & \bar{t} & -\underline{t} \end{bmatrix}^T
\end{aligned}
$$

a função Lagrangeana pode ser re-escrita como:

$$
\begin{aligned}
\mathcal{L}(x,u,\lambda,\pi) &= c_0 + c^T p_g + \tfrac{1}{2} p_g^T\, Q\, p_g + \lambda^T (p_L + B\,\theta - A_g\, p_g) \\
&\quad + \pi^T \left\{ [F_u\ F_\theta\ I] \begin{bmatrix} p_g \\ \theta \\ s \end{bmatrix} - L \right\} - \mu \sum_{i=1}^{n_g} (\ln\, \bar{s}_{gi} + \ln\, \underline{s}_{gi}) - \mu \sum_{i=1}^{n_l} (\ln\, \bar{s}_{ti} + \ln\, \underline{s}_{ti})
\end{aligned}
$$

Podemos agora escrever as condições de KKT :

$$
\begin{array}{rcccl}
\nabla_u \mathcal{L} &=& c + Q\, p_g - A_g^T\, \lambda + F_u^T\, \pi &=& 0 \\
\nabla_\theta \mathcal{L} &=& B^T\, \lambda + F_\theta^T\, \pi &=& 0 \\
\nabla_\lambda \mathcal{L} &=& p_L + B\,\theta - A_g\, p_g &=& 0 \\
\nabla_\pi \mathcal{L} &=& [F_u\ F_\theta\ I] \begin{bmatrix} p_g \\ \theta \\ s \end{bmatrix} - L &=& 0 \\
& & S\,\pi &=& \mu\, e
\end{array}
$$

Aplicando-se o método de Newton às condições de KKT obtem-se o seguinte sistema de equações linearizadas, que deve ser resolvido a cada iteração:

$$
\begin{aligned}
Q\ \Delta\, p_g - A_g^T\ \Delta\lambda + F_u^T\, \Delta\pi &= b_u^{(k)} \\
B^T\, \Delta\lambda + F_\theta^T\, \Delta\pi &= b_\theta^{(k)} \\
B\ \Delta\theta - A_g\, \Delta\, p_g &= b_\lambda^{(k)} \\
F_u\, \Delta\, p_g + F_\theta \Delta\theta + \Delta s &= b_\pi^{(k)} \\
S\, \Delta\pi + \Pi\, \Delta s &= b_s^{(k)}
\end{aligned}
$$

onde:

$$
\begin{aligned}
b_u^{(k)} &= -\,c - Q\,p_g^{(k)} + A_g^T\,\lambda^{(k)} - F_u^T\,\pi^{(k)} \\
b_\theta^{(k)} &= -B^T\,\lambda^{(k)} - F_\theta^T\,\pi^{(k)} \\
b_\lambda^{(k)} &= -\,p_L - B\,\theta^{(k)} + A_g\,p_g^{(k)} \\
b_\pi^{(k)} &= -\,[F_u\ F_\theta\ I]\begin{bmatrix} p_g^{(k)} \\ \theta^{(k)} \\ s^{(k)} \end{bmatrix} - L = \begin{bmatrix} -p_g^{(k)} - \bar{s}_g^{(k)} + \bar{p}_g \\ p_g^{(k)} - \underline{s}_g^{(k)} - \underline{p}_g \\ -\,\Gamma\,A\,\theta^{(k)} - \bar{s}_t^{(k)} + \bar{t} \\ \Gamma\,A\,\theta^{(k)} - \underline{s}_t^{(k)} - \underline{t} \end{bmatrix} \\
b_s^{(k)} &= \mu\,e - S^{(k)}\pi^{(k)}
\end{aligned}
$$

Após a eliminação de Δs o sistema reduzido torna-se:

$$
\begin{bmatrix} Q & 0 & -A_g^T & F_u^T \\ 0 & 0 & B^T & F_\theta^T \\ -A_g & B & 0 & 0 \\ F_u & F_\theta & 0 & -\Pi^{-1}\,S \end{bmatrix} \begin{bmatrix} \Delta p_g \\ \Delta\theta \\ \Delta\lambda \\ \Delta\pi \end{bmatrix} = \begin{bmatrix} b_u^{(k)} \\ b_\theta^{(k)} \\ b_\lambda^{(k)} \\ \widehat{b}_\pi^{(k)} \end{bmatrix}
$$